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New Method of Estimation of the Speed of Light in Vacuum Using Q-Values of Beta-Décay-Transitions in Mirrors Nuclei

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ABSTRACT

Purpose of the study: The goal of this work is to present a new method named Q-β-decay theory to estimate the speed of light. Our result is compared with the earlier estimations since the first measurement of Roemer in 1676 up to 2022, the latter being measured experimentally by picosecond laser technology.

Methodology: This research is based on an estimation of the speed of light in vaccum. This speed is expressed in terms of the *Q*-value of β^+- decay transitions in mirrors nuclei in the framework of the nuclear model with a nucleon constant density. The speed of light is calculated using the experimental *Q*-values of ^{37}K and ^{65}As miroirs nuclei.

Main Findings: The research gives the result *c* = 299 791 777 m/s agreeing excellently with the exact value 299 792 458 m/s with an accuracy at 0.000 23 %. This result shows that the speed *c* of light is obtained accurately from nuclear properties instead of light properties or electric and magnetic constants. A useful appendix for students details the methods of Roemer, Bradley, Fizeau, and Foucault along with the equations allowing one to calculate the speed of light they found.

Novelty/Originality of this study: Estimating the speed *c* of light with a high accuracy using a new method based on the properties of β^+- decay transitions in mirrors nuclei. As far as we know, it the first time that the speed of light is accurately estimated via a non-optical method 348 years ago since 1676.

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1. INTRODUCTION

During the Muslim Golden Age, the Arab scholars Al Kindi (801–873), Ibn Sahl (940–1000), Ibn al Haytham (965–1040) and Al Farisi (1267–1320), after rejecting the theories of extramission and intromission, discovered the phenomena of reflection and refraction of light moving in a straight line, at a high speed varying according to the medium through which it passed. In his research, Ibn Al-Haytham explains that light is material and propagates with a very high and finite speed according to rectilinear rays. The modification of this velocity by the bodies encountered causes refraction. The clash of light and bodies creates reflection [1].

During the Middle Ages, the Czech monk and physicist Ioannes Marcus Marci (1595-1667) and the Italian Francesco Maria Grimaldi (1618-1663) came to the conclusion that light was made up of waves propagating at high speed. After, the British Robert Hooke (1635-1703) admitted that light was composed of compression pulses propagating instantaneously or with a very high speed [2]. However, it was from 1676 that the Danish astronomer Olaus Roemer (1644-1710) made a series of observations of the eclipses of Io, a satellite of Jupiter, which he showed that the speed of light was finite and extremely great [3]- [10].

Roemer estimated for the first time, the speed of light at 214,000 km/s [4], [5], a result that overestimates up to 28.6% the accepted value of 299,792.458 km/s since 1983. In 1728, the British physicist James Bradley (1693-1762) estimated the speed of light in a vacuum, thanks to stellar aberrations, at about 301,000 km/s [4] with an accuracy of 0.40%. In 1849, the French physicist Hippolyte-Louis Fizeau (1819−1896) developed an optical device that allowed him to estimate the speed of light with good accuracy [4], [9, [11]. Fizeau then estimated the speed of light at 315,000 km/s [4], [11].

In addition, in 1862, the French physicist Léon Foucault (1819−1868) carried out an experiment at the Paris Observatory to measure the speed of light [12], [13] and estimated it at 298,000 km/s in air and at 225,000 km/s in water [8]. His result in air is obtained with an accuracy of 5.1%. After, the Polish-born American physicist Albert Abraham Michelson (1852-1931) attempted to increase the accuracy of the methods applied and succeeded in measuring the speed of light at 299,796 ± 4 km/s. This result is very accurate since it overestimates the exact value to only 0.0012%. The chronology of the famous experiments that provided the best estimates of the speed of light is as follows.

The English physicist Louis Essen (1908-1997) in collaboration with A.C. Gordon-Smith used a resonant cavity to estimate in 1947 the speed of light at 299 792 km/s ± 3 km/s with a high accuracy never obtained by optical measurements since 1900. Essen perfected his apparatus and increased the number of measurements up to 1950 and succeeded to achieve a very precise value equals to 299 792.5 km/s ± 1 km/s. This value was adopted at the 12th General Assembly of the International Radio-Scientific Union in 1957.

In 1949 Captain Carl I. Aslakson used a radar and found 299 792.4 ± 2.4 km/s. The English physicist Keith Davy Froome (1921-1995) used in 1958 a millimetre-wave radio interferometer instrument and found with high precision 299 792.50 ± 0.10 km/s. This measurement was the most accurate before the use of laser technology. In 1972, Kenneth Evenson (1932-2002) used stabilized helium-neon laser and found 299 792 456.2 m/s [14] with excellent accuracy at 0.000 000 2 %. In 1978, Woods, Shotton and Rowley used the same type of laser as Evenson but under stricter experimental conditions and found 299 792.45898 ± 0.0002 km/s with an excellent accuracy equals to 0.000 0003 %.

During the year 1983, the 17th General Conference on Weights and Measures adopted a new definition of the metre as follows: "*The metre is the length of the path travelled in a vacuum by light for a period of 1/299,792,458 second*". With this definition, the speed of light in vacuum is a universal constant with an exact value equals to 299 792 458 m/s. Although the exact value of the speed of light is fixed since 1983, researchers continue to actively perform new methods for the estimation of c.

Voudoukis [15] used a combination of simple, low-cost experiments and found the best value c = 292 960 000 m/s. Kushwaha and Adhikari [16] measured the speed of light from several light reflection on mirror and estimated it at 314 000 000 m/s. In addition, Arribas *et al*., [17] performed an indirect method to measure the speed of light. These authors measured the electrical permittivity ε_0 of air using a capacitance meter and the magnetic permittivity μ_0 of air using a solenoid and deduced the speed of light from the well-now relation $c^2\mu_0\varepsilon_0= 1$ as early described in the method of Maxwell who found in 1868 c = 284 200 000 m/s [18]. Arribas *et al*., [17] found $c = (3.00 \pm 0.06) \times 10^8$ m/s. Mehdi and Kolwankar [19] used laser beam and estimated the speed of light at $c = 2.95 \times 10^8$ m/s whereas Aljalal [20] used picosecond diode laser to determine the speed of light at $(2.996 \pm 0.002) \times 10^8$ m/s.

Besides with Updated Hubble Diagram of High-redshift Standard Candles including Type Ia Supernovae (SNe Ia) and high-redshift quasars (based on UV–X relation), Liu *et al*.,[21] measure the speed of Light an found for the SNe Ia Pantheon sample, combined with currently available sample of cosmic chronometers $c/c_0 = 1.03 \pm 0.03$ and for the Hubble diagram of UV+X-ray quasars acting as a new type of standard candle $c/c_0 = 1.19 \pm 0.07$, with c_0 = 299 792 458 m $\cdot$s^{-1} denotes the laboratory value of the speed of light. These last works show that calculations and measurements of the speed of light are very challenging.

In this paper, we aim to present a new non-optical method to estimate the velocity of light in vacuum. This new method is based on nuclear properties such as the β^+-decay process in mirrors nuclei $A = 2Z - 1$. In section 2 we give a brief description of the theoretical part of the work. In section 3 with present and discuss the results obtained and conclude in section 4.

2. BRIEF DESCRIPTION OF THE NEW Q- β⁺-DECAY THEORY

We recommend the reader to refer to [22] for further details on the theoretical development. Let us start from the The maximum energy $E_{\max}$ of the β^+ spectrum which is given by [22]

$$E_{\max} = \Delta W_{coul} - \left(m_n - m_p\right)c^2 - m_0 c^2 \quad ... (1)$$

In Eq.(1), ΔW_{coul} is the Coulomb energy loss during the considered isobaric transition, m_n and m_p denote respectively the mass of the neutron and the mass of the proton and m_0c^2 represents the electron rest mass energy.

Besides, in the framework of the nuclear model with a constant nucleon density, the radius R of the nucleus is $R = r_0A^{1/3}$ with r_0 the unit nuclear radius and A the mass number of the nucleus. In the later, AThe Coulomb energy of the charge nucleus $q = Ze$ distributed within the nuclear volume is given by the relation [22]

$$W_{coul} = \frac{3}{5}\frac{e^2}{R}Z^2 \quad \ldots (2)$$

In relation (2), the factor of 3/5 assumes the uniform volume distribution of protons in the nucleus. This factor is equal to 1/2 in the case of a surface distribution of the nuclear charge [22].

As soon as it is emitted, the positive charge of the particle β^+ is repelled by the nucleus. Coulomb repulsions thus occur near the emitting nucleus. Since the particle β^+ necessarily perturbs the distribution of protons in the nucleus during its emission, we hypothesize that the Coulomb energy loss during the isobaric transition is decreased by the mass energy m_0c^2 of the positron β^+. Not knowing the real distribution of the nuclear charge in the nucleus, we introduce a coefficient of nuclear distribution of protons within the nucleus that we denote $\alpha(Z)$.

The Coulomb energy given by Eq.(2) is then modified as follows for the parent ${}^{A}_{Z}X$ ($q = Ze$)

$$W^{\alpha}_{coul} = \alpha(Z)\frac{e^2}{R}Z^2 \quad \ldots (3)$$

In the case where all protons are uniformly distributed in the nuclear volume (absence of protons distributed on the surface of the nucleus), the coefficient of nuclear distribution of protons within the nucleus $\alpha(Z) = \alpha_0 = 3/5$.

For the daughter ${}^{A}_{Z-1}Y$ [$q = (Z - 1)e$] and taking into account the emission of the β^+ particle with rest energy mass m_0c^2

$$W^{\alpha}_{coul}(Y) = \alpha(Z)\frac{e^2}{R}(Z-1)^2 + m_0c^2 \quad \ldots (4)$$

Using Eqs. (3) and (4), we get $\Delta W_{coul} = W_{coul}(X) - W_{coul}(Y)$

$$\Delta W^{\alpha}_{coul} = \alpha(Z)\frac{e^2}{R}\left[Z^2 - (Z-1)^2\right] - m_0c^2 \quad \ldots (5)$$

That means

$$\Delta W^{\alpha}_{coul} = \alpha(Z)\frac{e^2}{R}(2Z-1) - m_0c^2 \ldots (6)$$

$$E^{\alpha}_{\max} = \Delta W^{\alpha}_{coul} - \left(m_n - m_p\right)c^2 - m_0c^2 \quad \ldots (7)$$

Using (4), Eq. (1) becomes

$$E^{\alpha}_{\max} = \alpha(Z)\frac{e^2}{R}(2Z-1) - \left(m_n - m_p\right)c^2 - 2m_0c^2 \quad \ldots (8)$$

For the mirrors nuclei $A = 2Z - 1$, Eq. (8) is in the final shape with $R = r_0A^{1/3}$

$$E^{\alpha}_{\max} = \alpha(Z)\frac{e^2}{r_0}A^{2/3} - \left(m_n - m_p\right)c^2 - 2m_0c^2 \quad \ldots (9)$$

3. RESEARCH METHOD

3.1 Expression of the speed of light in terms of the *Q*-value of mirror nuclei *A* = 2*Z* – 1

Let us then consider the first theoretical expression of the nuclear radius r_0 is given by the relation [23]

$$r_0 = \frac{\alpha^2}{2} a_0 \quad \dots (10)$$

With

- α the fine structure constant : $\alpha = \frac{e^2}{\hbar c}$
- a_0 the Bohr's radius : $a_0 = \frac{\hbar^2}{m_0 e^2}$

Carrying (10) into (9) and using the expressions of α et a_0, we find

$$Q = E_{max} + 2m_0 c^2 = 2\alpha(Z) m_0 c^2 A^{2/3} - (m_n - m_p)c^2 \dots \quad (11)$$

Eq. (11) is used to calculate the maximum energy of the β^+ spectrum of mirror nuclei $A = 2Z - 1$ if the coefficient of distribution of the nuclear charge within the nucleus is known. It also makes it possible to estimate the distribution of the nuclear charge within the nucleus by determining the $\alpha(Z)$ knowing Q since

$$2\alpha(Z) = \frac{Q + (m_n - m_p)c^2}{m_0 c^2 A^{2/3}} \dots (12)$$

To estimate the value of nuclear charge distribution coefficient, we neglect its dependency with the charge number Z. So Eq. (12) becomes

$$2\alpha_0 = \frac{Q + (m_n - m_p)c^2}{m_0 c^2 A^{2/3}} \quad \dots (13)$$

Eq.(13) is an equation with two unknowns α_0 and m_0c^2 (we intend in this work to calculate it directly from Q). Let us then establish an equation in terms of the nuclear charge distribution coefficient α_0. For that, we consider the following mass ratios from CODATA [24]

- Neutron-electron mass ratio: 1838.683 662 00;
- Proton-electron mass ratio: 1836.152 673 426

Using these data, with find

$$(m_n - m_p) = 2.530988574 m_0 = k m_0 \ ; \ k = 2.530988574.$$

Using (13) Eq. (14) is rewritten as follows

$$2\alpha_0 = \frac{Q + k m_0 c^2}{m_0 c^2 A^{2/3}} \qquad \dots (14)$$

That means

$$Q = \left(2\alpha_0 A^{2/3} - k\right) m_0 c^2 \dots (15)$$

So finally

$$m_0 c^2 = \frac{Q}{\left(2\alpha_0\ A^{2/3} - k\right)} \quad \ldots (16)$$

Let us now consider two mirrors nuclei ${}^{x}X_1$ ($x = A_1$) et ${}^{y}X_2$ ($y = A_2$) with $x > y$. We get from Eq.(16)

$$\begin{cases} m_0 c^2 = \dfrac{Q(x)}{\left(2\alpha_0\ x^{2/3} - k\right)} \\ m_0 c^2 = \dfrac{Q(y)}{\left(2\alpha_0\ y^{2/3} - k\right)} \end{cases} \Rightarrow \frac{Q(x)}{\left(2\alpha_0\ x^{2/3} - k\right)} = \frac{Q(y)}{\left(2\alpha_0\ y^{2/3} - k\right)} \quad \ldots (17)$$

Using (17), we express finally the nuclear charge distribution coefficient α_0 in terms of the Q-value.

$$2\alpha_0 = \frac{k\left[Q(x) - Q(y)\right]}{\left[Q(x)y^{2/3} - Q(y)x^{2/3}\right]} > 0 \ \ldots (18)$$

For various mirrors nuclei, primary calculations indicated that ^{37}K and ^{65}As have the same nuclear charge distribution coefficient. From [25] we extract the Q-value experimental data.

- For ^{37}K: $Q(y) = 6.1475$ MeV; $y = 37$;
- For ^{65}As: $Q(x) = 9.5400$ MeV; $x = 65$.

Using these experimental data, we get from Eq. (18) taking into account the value of k given equals to 2.530988574

$$2\alpha_0 = \frac{2.530988574 \times \left[9.5400 - 6.1475\right]}{\left[9.5400 \times 37^{2/3} - 6.1475 \times 65^{2/3}\right]} = 1.311420475425.$$

Remarking that 3/5 = 0.6 and $\alpha_0 = 0.655710$, we obtain approximately

$$\begin{cases} 2\alpha_0 = \dfrac{6}{5} + 0.1114 = 1.3114 \\ \alpha_0 = \dfrac{3}{5} + 0.0557 = 0.6557 \end{cases} \ldots (19)$$

4. RESULTS AND DISCUSSION

Using Eq. (15), we get

$$c = \sqrt{\frac{Q}{\left(2\alpha_0\ A^{2/3} - k\right)\ m_0}} \quad \ldots (20)$$

The average value of the velocity c of light in vacuum is calculated from Eq. (39) using: $\alpha_0 = 0.6557$, $k = 2.530988574$ and $A = 37$ and 65 respectively for ^{37}K ($Q = 6.1475$ MeV) and ^{65}As ($Q = 9.5400$ MeV). In addition, $m_0 = (9.109\ 383\ 7139 \pm 0.000\ 000\ 0028) \times 10^{-31}$kg and 1 eV = $1.602\ 176\ 634 \times 10^{-19}$ J [24]. We obtain then

$$\begin{cases} c = \sqrt{\dfrac{6.1475\times10^{6}\times1.602176634\times10^{-19}}{\left(1.3114\times37^{2/3}-2.53098857\right)\times9.1093837139\times10^{-31}}} = 299791886.823 \\ c = \sqrt{\dfrac{9.5400\times10^{6}\times1.602176634\times10^{-19}}{\left(1.3114\times65^{2/3}-2.53098857\right)\times9.1093837139\times10^{-31}}} = 299791668.220 \end{cases}.$$

The average value obtained is

c = 299 791 777 m/s. (21)

The result (21) is in excellent agreement with the accepted value equal to 299,792,458 m/s (CODATA, 2022) with an accuracy of 0.000 23% [100 × (299 792 458 – 299 791 777)/299 792 458 = 0.000227%]. Table 1 summarizes comparison of the present calculations with the earlier optical methods from 1676 to 1947

Table 1. Important dates, researchers and experimental methods relating to the main attempts to estimate the speed of light from 1676 to 1947

Date	Researchers	Method	Speed (km/s)*	Accuracy **
1676	Olaus Roemer	Satellites of Jupiter	214 000	28.67 %
1726	James Bradley	Stellar aberration	301 000	0.40 %
1849	Armand Fizeau	Toothed wheel	315 000	5.07 %
1862	Léon Foucault	Roting Miror	298 000	0.60 %
1879	Albert Michelson	Roting Miror	299 910	0.04 %
1907	Edward Bennett Rosa & Noah Earnest Dorsey	Electromagnetic Constants	299,788	0.0014%
1926	Albert Michelson	Roting Miror	299 796	0.0012 %
1947	Louis Essen et coll.	Resonant cavity	299 792	0.000 15
2024	Ibrahima Sakho	β-decay theory	299 792,0	0,000 15

* Results taken from GIBBS [4] ;
** With respect to the exact value c = 299 792. 458 km/s.

As shown in Table 1, the present method gives a result with the same uncertainty than the measurement of Essen and one of the best estimation between 1676 and 1947. Table 2 show a comparison between the present calculations and very recent optical methods from 2018 to 2022.

Table 2. Important dates, researchers and experimental methods relating to the main attempts to estimate the speed of light from 2018 to 2022.

Date	Researchers	Method	Speed (km/s)	Accuracy *
2018	Voudoukis	Light interference or diffraction	292 960	2.28 %
2020	Kushwaha and Adhikari	Several light reflection on mirror	314 000	4.74 %
2020	Arribas *et al.*,	Electrical permittivity and magnetic permittivity	300 000	0.07 %
2021	Mehdi and Kolwankar	Laser beam	295 000	1.60 %
2022	Al-Jalal	picosecond diode laser	299 600	0.06%
2024	Present calculations	β-decay theory	299 792	0.000 15

*with respect to the exact value c = 299 792. 458 km/s.

In table 2, comparison indicates that the present formalism provides the most accurate result to date. Measurement uncertainty on Q-value may constraint our relative error. However, research is underway to perfect the formalism in order to increase the precision from 0.00015% to 0.000 000 1% knowing that the most precise current measurements are those obtained by Evenson, in 1972 with a precision of 0.000 000 2 % [14]. Overall, the present study is very original as it the first time that the speed of light is accurately estimated via a non-optical method since the first estimation of Roemer in 1676.

5. CONCLUSION

In this paper, we have estimated via a novel methodology, the velocity *c* of light in vacuum with a percentage deviation at 0.000 23 % relatively to the exact value. From the first measurement by Roemer in 1676 until 1983, when the 15th General Conference on Weights and Measures set the speed of light at 299,792,458 m/s, all the methods that were used to estimate the speed of light *c* exploited the properties of light itself. For the first time, the speed of light is estimated by a method exploiting the properties of radioactive nuclei. Knowledge of the law of variation of the nuclear distribution coefficient α (Z) would make it possible to revise the present accuracy and to make the β-decay theory one of the famous methods that have made it possible to determine the speed of light with the best precision. Research is being carried out in this direction. A useful document for students is added in the appendix section where the methods of Roemer, Bradley, Fizeau, and Foucault are explained in details along with the equations allowing one to calculate the speed of light they found.

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APPENDIX

OVERVIEW OF SOME EXPERIMENTAL METHOD USED IN THE PAST TO ESTIMATE THE VELOCITY OF LIGHT

In this section, we consider only the experiments for which, the velocity of light is well expressed and directly calculable. This appendix is a useful document for students and writers focusing their activities on the history of the main methods used to estimate the speed of light since the first measurement by Roemer in 1676. According to what is available in the literature, we'll then consider chronologically the experiments of Roemer, Bradley and Fizeau. These experiments are detailed below.

I. FIRST ESTIMATE OF THE SPEED OF LIGHT BY OLAUS ROEMER (1676), OBSERVATION OF THE ECLIPSES OF JUPITER'S SATELLITE IO

A few years after the invention of the telescope and after making some minor improvements to it, the Danish astronomer Olaus Roemer (1644-1710) was the first scientist to measure the speed of light in 1676 [4] by a series of observations of the eclipses of Io, a satellite of Jupiter. Roemer worked at the Paris Observatory led by the French astronomer Jean-Dominique Cassini (1625-1712). Realizing that the distance between Earth and Jupiter varied, due to the orbital trajectories of the planets, Roemer gave the first proof of the finite nature of the speed of light. The diagram that Roemer had imagined is shown in Figure 1.

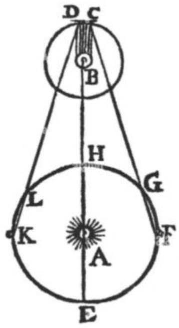

Figure 1. Diagram accompanying Roemer's dissertation. Immersions are observed when the Earth approaches Jupiter, going from F to G. Emersions are observed when the Earth moves away from Jupiter, going from L to K [3].

Figure 2 gives a representation of Roemer's diagram in the particular case of immersions observed when the Earth moves away from Jupiter, going from *L* to *K*.

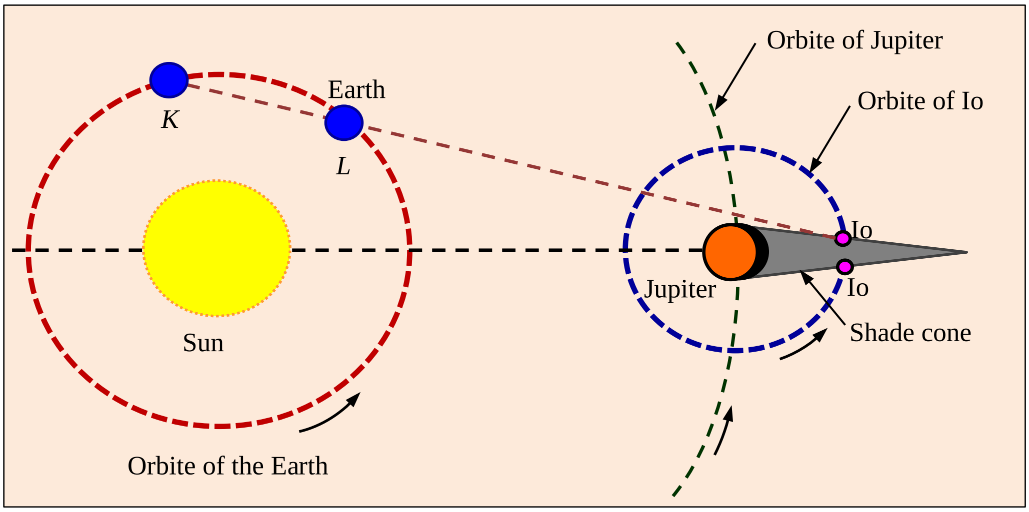

Figure 2. Representation of Roemer's diagram in the particular case of the immersions observed when the Earth moves away from Jupiter, going from L to K (image designed by the author).

As shown in Figure 2, Roemer observed the eclipse of Jupiter's satellite Io. In this figure, we see that the orbit of Io entering Jupiter's shadow cone (immersion) and exiting it (emersion). We also see the Sun and the Earth's orbit. Between two successive emersions of Io, the Earth moves from *L* to *K*. When the Earth is in *K*, Roemer observed the second emersion of Io several minutes late since the light from Io has travelled the extra distance *LK*.

On the basis of his observations, Roemer assumed that the time interval between two successive eclipses is constant and equal to the time of one revolution of the satellite around Jupiter. Using the distance Earth-Sun calculated by Cassini, Roemer determined the speed of light who took 22 minutes to pass through the diameter of the Earth's orbit. Let us first specify the method of calculating the Earth-Sun distance calculated by Cassini. This method uses the notion of parallax illustrated in Figure 3. By definition, parallax is the difference between the apparent positions of an object when observed along two lines of sight. Parallax is easily demonstrated by holding a pencil in front of you. When you alternately close one eye and then the other, the position of the pencil seems to change in relation to distant objects in the background (fig.3).

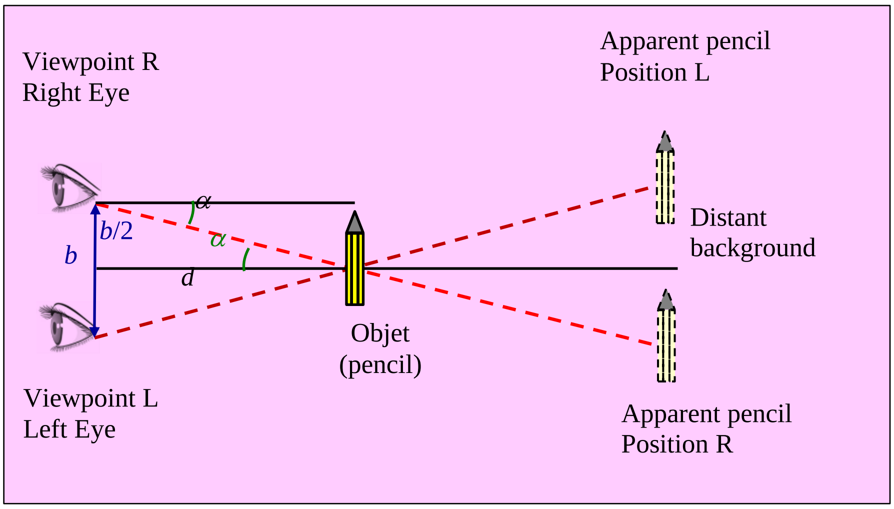

Figure 3. Schematic illustration of the notion of parallax. Two lines of sight are possible when one eye (right eye, position R) is closed alternately and then the other (left eye, position KL), the position of the pencil seems to change in relation to distant objects. The distance between the two eyes (R and L positions) is not respected for more clarity (image designed by the author).

Since there is a distance between the two eyes, each eye presents a different view of the pencil in relation to the background. The brain processes both views simultaneously. Measurement of the parallax makes it possible to estimate distances.

In Figure 3, the parameter b called the baseline, represents the distance between the two points of view of the object. The angle α is the parallax angle. The distance d from the observed object is determined by the measurement b and α. Applying the properties of the right triangle, one obtains:

$$d = \frac{b}{2\tan\alpha} \quad \dots (1)$$

To calculate the distance from the Earth to the Sun, Cassini had the presence of mind to exploit an opportunity offered to him in 1672.

Indeed, many astronomers knew that Mars and Earth would be in opposition in September and October next 1672. In addition, the English astronomer John Flamsteed (1646-1719) predicted that Mars will pass in front of a particular star in the constellation Aquarius (the middle Psi star) on October 1, 1672. Cassini exploited the prediction regarding the probable opposition of Mars and Earth. When the Sun, Earth and Mars are in a straight line (fig.4), the distance from the Earth to Mars (a_{EM}) is determined along with the value of the astronomical unit (AU).

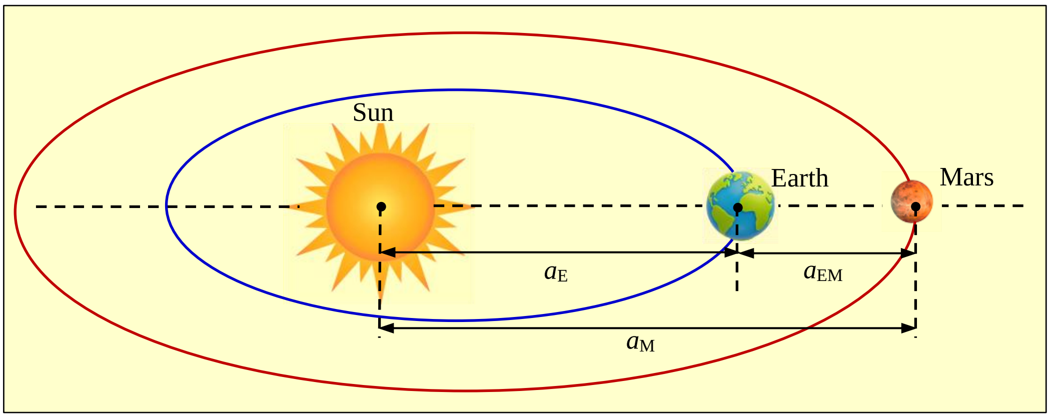

Figure 4. Special situation of the Earth and the planet Mars in opposition to the Sun (image conceived by the author). The dimensions of the stars are not respected, as are the lengths of the semi-major axis with elliptical orbits (image designed by the author).

Note that the opposition of Mars occurs when the Earth hides Mars and positions itself between the Sun and Mars. Every 15 to 17 years, we witness very close oppositions of Mars as the Earth passes between Mars and the Sun at the perihelion of Mars [10].

In Figure 4:

- a_E is the Earth-Sun distance equal to 1 AU by definition;
- a_M is the distance from Mars to the Sun;
- a_{EM} is the distance from Earth to Mars.

The three celestial objects being aligned, we obtain the simple relationship:

$$a_M = a_E + a_{EM} \quad \ldots (2)$$

The Earth-Mars distance is calculated using Kepler's third law. According to this law, the ratio of the square of the period of revolution T of a planet to the cube of the semi-major axis a of its orbit is constant, i.e.:

$$\frac{T^2}{a^3} = Cte \quad \ldots (3)$$

The constant Cte is independent of the mass of the planet (it depends only on the mass of the Sun creating the gravitational field in which the planet is moving).

Using (3), we obtain for the Earth and for Mars:

$$\frac{T_E^2}{a_E^3} = \frac{T_M^2}{a_M^3} \Rightarrow a_M = \left(\frac{T_M}{T_E}\right)^{2/3} a_E \quad \ldots (4)$$

The period of revolution of the Earth is equal to 1 year: T_E = 1 year and its distance from the Sun a_E = 1 AU. A year on Mars lasts about 687 Earth days, which corresponds to 1.88 Earth years (687/365.25). Using (4), we determine the Sun-Mars distance a_M, i.e.:

$$a_M = (1{,}88)^{2/3} \times 1 = 1.5232 \text{ UA} \approx 1.52 \text{ UA}. \quad \ldots (5)$$

Using (2), we draw the distance from Earth to Mars:

$$a_{EM} = a_M - a_T = 0.52 \text{ UA}. \ldots (6)$$

To determine the distance of the Earth from the Sun, Cassini undertook between 1672 and 1673, the measurement of the parallax of Mars. For this, he collaborated with the astronomer Jean Richer (1630-1696). The great circle distance from Cayenne to Paris, measured on the Earth's surface, is 7,089 km. However, the distance required for Cassini's calculations is in fact the cord distance from Cayenne to Paris which is about 6,700 km [9], [26], [27]. The principle of measuring the parallax of Mars is illustrated in Figure 5.

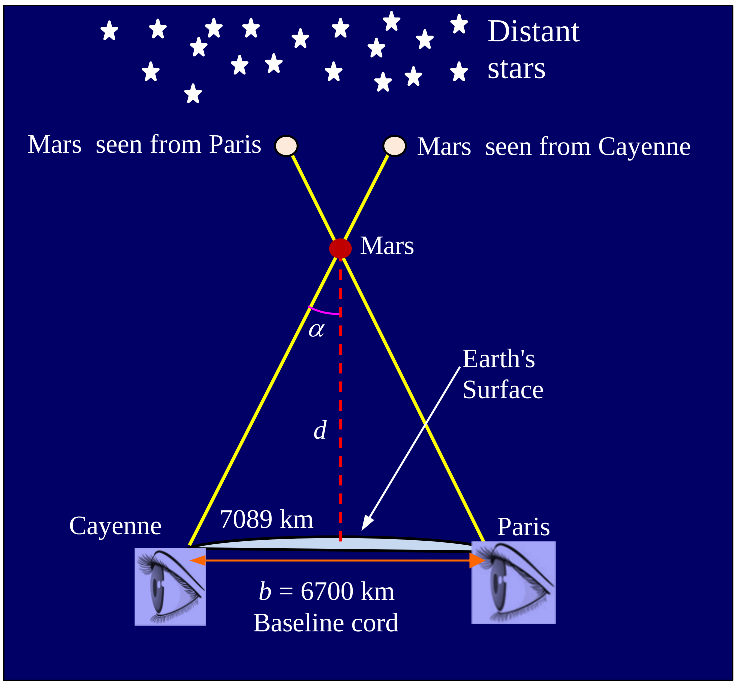

Figure 5. Principle of measurement of the parallax of Mars by Cassini. Seen from Paris or Cayenne at the same time, the position of Mars in relation to the stars is slightly different (parallax phenomenon). The baseline b is equal to the cord distance between Cayenne and Paris (image designed by the author).

Cassini and his team found the parallax angle to be $\alpha = 9.5''$ [9]. Now, one arcseconds (1") is equal to 1/60 of an arc minute, i.e. 1/3600 of a degree. So

$$\alpha = 9.5'' \approx 0.0026389° \quad \ldots(7)$$

Using (7) and the data in Figure 5, equation (1) gives the Earth-Mars distance $a_{EM} = d$, i.e.

$$d = \frac{6700}{2 \times \tan 0.0026389} = 72{,}735{,}178.0034 \text{ km.}$$

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$$a_{EM} = 72{,}735{,}178 \text{ km.} \qquad \ldots (8)$$

Note that the closest recorded distance between Mars and Earth was recorded in August 2003, when the two planets were 34.8 million miles (56 million kilometers) apart. According to NASA, the two planets will no longer be as close as they were in 2237 [10]. The result (8) dated in 1673 overestimates the accepted value at 56,000,000 km by about 30%.

Using (6) and (8), we convert the astronomical unit into kilometers, i.e.

$$a_{EM} = 72{,}735{,}178 \text{ km} = 0.52 \text{ UA} \Rightarrow 1 \text{ AU} = 139{,}875{,}342 \text{ km.}$$

The distance from the Earth to the Sun measured by Cassini in 1673 is then equal to

$$a_E = 139\,875\,342\,000 \text{ m} \quad \ldots(9)$$

The accepted value of the Earth-Sun distance is equal to 149,597,870,700 m [28]. Cassini's measurement thus overestimates the average distance from the Earth to the Sun by 6.5%.

Let us now move on to Roomer's calculation of the speed of light

As explained above, Roemer established that light needs a time t = 22 min to cross the diameter of the Earth's orbit in a straight line. The radius of the Earth is estimated to be equal to the value of the semi-major axis a_E. So the Earth's diameter is $2a_E$. The expression for the speed of light is then given by

$$c = \frac{2a_E}{t} \qquad \dots (10)$$

Digital application: a_E = 139,875,342 km, t = 1,320 s

$$c = \frac{2 \times 139{,}875{,}342}{1{,}320} = 211{,}932.33\,\text{km/s}.$$

Thus, in 1676, Roemer estimated the speed of light for the first time at [9]

$$c = 211{,}932 \text{ km/s} = 211,\ 932,\ 000 \text{ m/s} \qquad \dots (11)$$

The result (11) overestimates the accepted value (299,792,458 m/s) by 29%. This lack of precision is due in particular to Cassini's overestimation of the Earth-Sun distance at 6.5%. Two errors are responsible for most of the difference between the speed calculated by Roemer and its exact value. Firstly, the light travels through Earth's orbit about 16.6 minutes instead of 22 minutes. Secondly, the distance between the Earth and the Sun is 149,600,000 km instead of the 139,875,342 measured by Cassini [9]. However, many authors have pointed out that, contrary to popular belief, Roemer did not calculate the speed of light [6, 7]. Some authors specify that Roemer estimated the speed of light at 214,000 km/s [4], [5]. Others claim that the credit for proving that the speed of light is not infinite goes to Cassini and Roemer. BOBIN *et al.*, [6] indicate that, Cassini and Roemer did not give a numerical value. Christiaan Huygens (1629−1695) who was also at Paris Observatory, made a calculation reproduced in his Treatise on Light in 1690 and claimed that "the speed of light is more than 600,000 times greater than the speed of sound" (in fact 660,000 times with his data). In addition, Michel Bougard points out in their collective work [8], "... neither Rømer nor Cassini sought to go further in calculating the speed of light. Roemer was therefore not the first to "measure" this speed. It was Huygens who determined the speed of light at 212,395 km/s based on Rømer's hypothesis and the measurements made by Abbé Jean Picard (1620-1682). The numerical value at 220, 000 km/s according to the available data, was however not published by Huygens until 1690 in his treatise on light" [29].

2.2. Estimation of the speed of light by James Bradley (1728), observation of stellar aberrations

In 1728, the British physicist James Bradley (1693-1762) estimated the speed of light in vacuum, thanks to stellar aberrations, at about 301,000 km/s. These aberrations are manifested by a visible shift in the position of stars due to the motion of the Earth around the Sun. The degree of stellar aberration can be determined from the ratio of the Earth's orbital velocity to the speed of light. By measuring the angle of the stellar aberration θ and by applying this data to the orbital velocity of the Earth (fig.6), Bradley was able to accurately estimate the speed of light.

As shown in Figure 6, the star appears in its true position when the Earth is stationary relative to the star. However, as the Earth moves in its orbit, the star's position changes, appearing ahead of its true position. In December and June, the star's apparent position is close to its true direction. However, in March and September, there is a noticeable angle between the actual position of the star and its apparent position. The angle of aberration θ measures th*e change in the apparent position of the star due to the motion of the Earth*. This angle between the actual and apparent position of the star depends on the speed of the star. Knowing θ and the orbital speed v of the Earth, the speed of light is given by [9]

$$c = \frac{v}{\theta}. \qquad (12)$$

In equation (12), θ is expressed in radians (not in degrees).

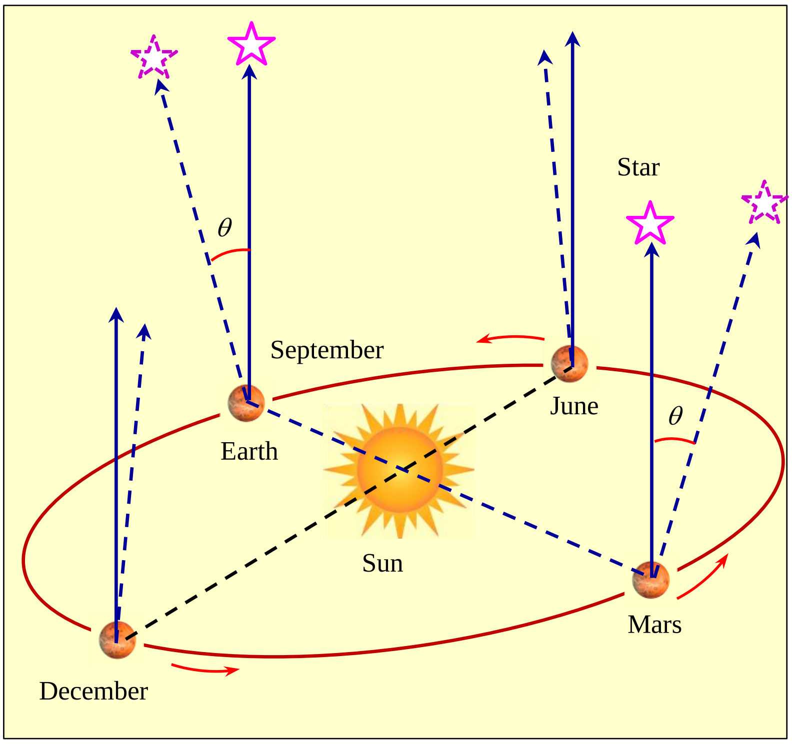

Figure 6. A technique used by Bradley to measure the speed of light. The arrows in solid lines indicate the actual positions of the star in the month of observation while the dotted arrows indicate the corresponding apparent positions (image designed by the author).

Bradley's measurements give 20 arcseconds (20″). Moreover, in the time of Bradley, the orbital speed of the Earth was v = 29.2 km/s. For conversion: 1° = 3600 arcseconds; 2π radian = 360°. So we get

$$\theta = \frac{20}{3600} \times \frac{2\pi}{360} = 9.6962736 \times 10^{-5} \text{ Radian}$$

Using (12), we obtain

$$c = \frac{29.2}{9.6962736 \times 10^{-5}} = 301146.617 \text{ km/s}$$

The value of the speed of light estimated by Bradley in 1728 is then equal to [30]

c = 301 000 km/s or 301 000 km/s. (13)

Bradley's measurement (13) overestimates the exact value (299,792,458 km/s) by 0.4%.

2.3. Estimation of the speed of light by Hippolyte-Louis Fizeau (1849), cogwheel experiment

The first terrestrial measurement of the speed of light was recorded by the French scientist Armand Fizeau (1819-1896) in 1849 [30]. Fizeau developed an optical device that allowed him to estimate the speed of light with good precision. The equipment of his experimental device is the following

1 – Light source
2 – Translucent beam splitting mirror
3 – Moving cogwheel, light beam breaker
4 – Remote mirror;
5 – Eyepiece.

Figure 7 shows the experimental set-up of Fizeau's cogwheel.

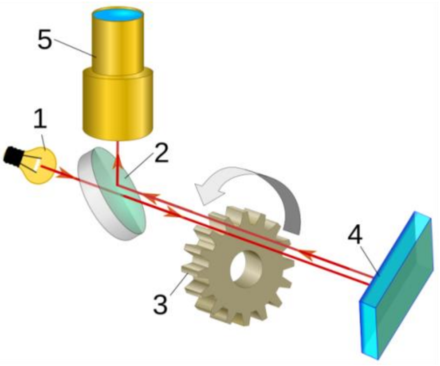

Figure 7. Diagram of the Fizeau experiment to determine the speed of light (https://fr.wikipedia.org/wiki/Hippolyte_Fizeau, 2024).

Let us describe the principle of the Fizeau experiment. For this purpose, we consider the simplified diagram shown in Figure 8.

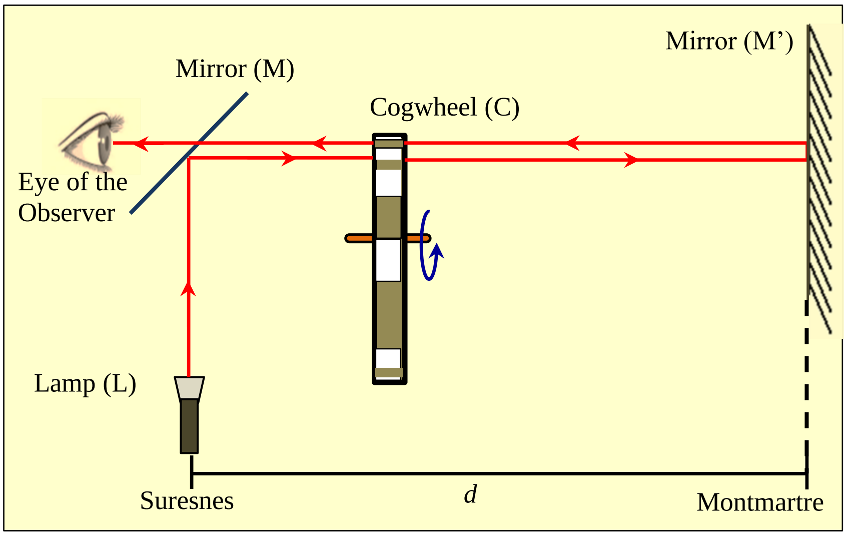

Figure 8. Simplified diagram of Fizeau's experimental setup (image designed by the author)

In Suresnes (next to La Défense), a lamp (L) illuminates a mirror (M) without a tint inclined at 45°. The light beam therefore passes just through the teeth of a toothed wheel (C) containing 720 teeth, motionless, before moving away, and hitting a mirror (M') located at a distance d = 8633 m from Montmartre. The light then goes back in the other direction, crosses the wheel again by passing just between two teeth, and finally passes through the mirror (M), where an observer can see it. The gear is rotated by the motor driven by a weight. The light travels the distance 2d which corresponds to the Suresnes/Montmartre/Suresnes route. Fizeau interposed a cog wheel rotating at a known speed (12.68 revolutions per second) between the light source and a the mirror (M'). His device was installed so that the light wave propagates between the belvedere of his house in Suresnes and the window of a house in Montmartre, a round trip distance of some 17 kilometers. With his device, Fizeau estimated the speed of light at 315,000 km/s [4, 11, 30].

Let us now determine the value of the speed of light c in accordance with Fizeau's experiment (fig. 8). To do this, we designate by

- d: the distance between Suresnes and Montmartre
- ω : the angular speed of rotation of the wheel at the moment when the light disappears
- n: the number of teeth of the wheel.

Data: d = 8633 m; average speed found by Fizeau**:** ω = 12.68 tr/s ; n = 720.

The time needed for the light to make the round trip from Suresnes to Montmartre is

$$t = \frac{2d}{c} \qquad \ldots(14)$$

The wheel contains *n* teeth but also *n* hollows, i.e. a total of 2*n* teeth and hollows. When the wheel (driving the 2*n* teeth and hollows) makes a revolution corresponding to an angular-axis $\theta = 2\pi$, each tooth or hollow scans an elementary angle

$$\delta\theta = \frac{\theta}{2n} = \frac{\pi}{n} \qquad \ldots (15)$$

The hourly equation of a tooth in uniform rotational motion at speed ω is

$$\delta\theta = \omega t \qquad \ldots(16)$$

When the light travels back and forth from Suresnes to Montmartre, a tooth and a hollow will have rotated for a duration *t* equal to

$$t = \frac{\delta\theta}{\omega} = \frac{\pi}{n\omega} \qquad \ldots(17)$$

By equalizing (14) and (17), we derive the expression of the speed of light

$$c = \frac{2n\omega d}{\pi} \qquad \ldots.(18)$$

Digital application (the unit of an angular velocity is in radians per second. 1 round corresponds to 2π. So for 12.68 tours we get $\omega = 2\pi \times 12{,}68$ rad/s).

$$c = \frac{2 \times 720 \times 2\pi \times 12.68 \times 8633}{\pi} = 315{,}263{,}347.2 \text{ km/s.}$$

The speed of light estimated by Fizeau in 1849 is equal to [4], [11], [30]

c = 315,000 km/s. (19)

The result (19) overestimates the exact value (299,792,458 km/s) by 4.5%. It was improved by Léon Foucault by using rotating mirrors, which gave the much more accurate value of 298,000 km/s (with an accuracy of 0.6%). His technique was good enough to confirm that light travels more slowly in water than in air [4]. Finally, it should be mentioned that Fizeau was decorated with the Legion of Honor. Even if his result lacks precision (4.5%), Fizeau was the first to have installed a device capable of following the path of light and measuring its speed on earth. It should be underlined that Bernard Pire [11] mentioned in his article that, Fizeau interposed a cogwheel rotating at 12.68 revolutions per second. Others, mention that Fizeau measured an average angular speed at 12.6 revolutions per second [9]. In the latter case, equation (18) gives the result *c* = 313,274,304 km/s.

Fizeau's experiment was later modified by French physicist Jean Léon Foucault (1819-1868), who replaced the toothed wheel with a rotating mirror and found 298,000 km/s. The Foucault method was further improved by many up to 1927, with the most precise measurement being made by Albert A. Michelson (1852-1931) [30]. The average result of a large number of measurements done by Michelson in 1927 was 299,796 ± 4 km/s [31].