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Research Article

GASING Method Training and Elementary Teachers' Content Knowledge of Basic Arithmetic in West Manggarai, Indonesia

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ABSTRACT

Purpose of the study: This study examined the effectiveness of GASING (*Gampang, Asyik, dan Menyenangkan*) method training on elementary school teachers' content knowledge of basic arithmetic operations, namely addition, subtraction, multiplication, and division, in West Manggarai Regency, East Nusa Tenggara, Indonesia.

Methodology: A pre-experimental one-group pretest-posttest design was employed. Fifty-five elementary school teachers in West Manggarai Regency participated in a 10-day (80-hour) GASING training program. A content knowledge test (Cronbach's alpha = 0.89) was administered as the measurement instrument. Data were analyzed using the Wilcoxon signed-rank test, Hake's normalized gain (N-gain), and Cohen's d, processed with SPSS version 26.

Main Findings: Teachers' content knowledge improved substantially following training, with mean scores rising from 31.00 to 72.09 out of 100, more than double the pretest level. This gain was statistically significant (Wilcoxon $Z = -6.441$, $p < 0.001$) and represented a very large practical effect (Cohen's $d = 2.94$), indicating the improvement was not only reliable but also substantively meaningful. Normalized gain analysis (N-gain = 0.59, medium category) showed that 96.4% of teachers achieved moderate-to-high learning gains, suggesting the training was effective across nearly the entire cohort rather than only for a subset of participants.

Novelty/Originality of this study: This study extends the existing GASING literature, unlike prior studies that centered on student learning outcomes or general teacher motivation, this study contributes targeted evidence on teacher professional development in basic arithmetic instruction, offering a replicable, low-resource training model applicable to similarly under-resourced regions in eastern Indonesia.

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1. INTRODUCTION

Teachers' content knowledge is a fundamental pillar of quality mathematics instruction. Ball et al. [1] defined mathematical knowledge for teaching (MKT) as a specialized understanding encompassing mathematical concepts, inter-conceptual relationships, and multiple representational strategies, and this framework guides the present study's conceptualization of teacher content knowledge as a domain distinct from, yet foundational to, effective mathematics pedagogy. Hill et al. [2], [3] demonstrated that this knowledge directly and substantially elevates the quality of instruction and student achievement, underscoring the need to strengthen content competency among elementary teachers.

In Indonesia, this need is particularly urgent: the 2022 PISA results ranked Indonesian students 65th of 81 countries, with a mean mathematics score of 366, significantly below the OECD average of 472 [4], and only 18% reached at least Level 2 proficiency compared to 69% internationally [5]. Wijaya et al. [6] identified inadequate teacher training as a primary driver of this performance, prompting Kemdikbudristek [7] to call for systematic intervention to strengthen teachers' competency in basic arithmetic, the foundation of elementary mathematics instruction.

These challenges intensify in regions with limited educational infrastructure, such as West Manggarai Regency, East Nusa Tenggara. The World Bank [8] documented structural barriers including limited infrastructure and geographic isolation in such regions; Fedi et al. [9] found a substantial gap between local (Manggarai) cultural mathematical understanding and teachers' formal mathematical competency; and Utami et al. [10] showed that teachers here require adaptive, contextually responsive training to overcome restricted access to conventional professional development. Puspitasari et al. [11] further demonstrated that effective curriculum implementation in such resource-limited settings requires strategic, adaptive management balancing accountability with instructional flexibility.

The GASING (Gampang, Asyik, dan Menyenangkan) method, developed by Yohanes Surya, addresses this gap through a three-stage concrete–abstract–mental arithmetic progression [12], [13], enabling learners to construct arithmetic understanding through active, staged exploration with concrete materials [14] rather than conventional procedural instruction. Within the MKT framework, this concrete-to-abstract progression is theoretically positioned to build the deep conceptual understanding, rather than mere procedural fluency, that Ball et al. [1] identify as central to teachers' specialized content knowledge. The method's critical points, addition <20, multiplication 1–10, subtraction <20, and division ≤100, align with the internationally validated Concrete-Representational-Abstract (CRA) framework [15], [16], which a meta-analysis by Carbonneau et al. [17] found yields a mean effect size of 0.55 for concrete manipulatives in mathematics learning.

Prior GASING research consistently demonstrates its effectiveness for student learning across diverse contexts and populations, ranging from matriculation students [18] and rural learners [19] to students with intellectual disabilities [20], general elementary problem-solving [21], online delivery formats [22], and integrated drill-based interventions in Eastern Indonesia yielding an N-Gain of 0.7964 [23]. However, Wijiningsih et al. [24], auditing 83 empirical GASING studies, found that 78.3% focus exclusively on cognitive outcomes. Notably, this body of evidence exhibits three specific limitations that constitute the gap addressed by the present study: no prior study has quantified the effect size of GASING training on adult learners such as teachers; none has applied Hake's normalized gain, a measure standard in teacher-training evaluation, to GASING outcomes; and none has examined GASING's effectiveness for elementary teachers' own content knowledge, as distinct from their capacity to deliver GASING instruction to students, particularly in regions with limited professional development access. This gap is theoretically significant because effective mathematics teacher professional development must simultaneously enhance content and pedagogical understanding [25], consistent with the MKT framework's premise that content knowledge and pedagogical knowledge are interdependent rather than separable domains. Fukaya et al. [26] found teachers' pedagogical content knowledge (PCK) correlates with teaching motivation, while the TEDS-M study [27] revealed that elementary teachers in developing countries generally hold content knowledge below international standards.

Teachers in regions with limited access to professional development face compounding structural barriers to building content knowledge. Hassler et al. [28] found only 60–70% of teachers in isolated regions show substantial improvement through conventional training, while Tamur [29] identified limited time, materials access, and technology as primary constraints on teacher PD specifically within East Nusa Tenggara, positioning intensive manipulative-based models like GASING as a contextually relevant solution. This pattern is corroborated more broadly across similarly under-resourced settings by Tanujaya et al. [30], Dwiyono and Tasik [31], and Rahmatin and Marzuki [32], who each found that limited training access directly constrains teachers' conceptual understanding.

Kennedy [33] and Desimone [34], [35] established that effective professional development requires adequate time, active learning, content focus, coherence, and collective participation, while Darling-Hammond et al. [36] and Garet et al. [37] confirmed that intensive, hands-on training produces substantial impact. Collectively, these theoretical and empirical strands, MKT as the conceptual lens, CRA as the pedagogical mechanism, and the documented professional development needs of under-resourced regions, converge to suggest that the GASING training model, with its structured progressive stages, concrete manipulatives, and intensive 10-day format, is well-positioned to meet the professional development needs of elementary teachers in West Manggarai Regency. Accordingly, this study examined the effectiveness of GASING method training in improving elementary teachers' content knowledge of basic arithmetic in West Manggarai Regency, East Nusa Tenggara, specifically measuring: (1) the magnitude of improvement before and after training; (2) the improvement category based on Hake's normalized gain; and (3) the Cohen's d effect size of the intervention.

2. RESEARCH METHOD

This study employed a pre-experimental one-group pretest-posttest design [38]. This design was selected based on practical and ethical considerations, given the limited access to professional development in the study location, which precluded the formation of a control group. The research participants consisted of 55 elementary school teachers in West Manggarai Regency, East Nusa Tenggara, who completed the Comprehensive Numeracy Program based on the GASING method in its entirety. The participants were drawn from various elementary schools across the regency, which are characterized by limited access to professional development programs [8], [39]. Inclusion criteria were as follows: (1) currently teaching in Grades 1-6 in Grades 1–6; (2) willingness to participate in the full 10-day intensive training program; and (3) full completion of both the pretest and posttest.

The research instrument consisted of a content knowledge test specifically designed to measure teachers' conceptual and procedural understanding of basic arithmetic operations, encompassing: (1) addition of whole numbers; (2) multiplication of whole numbers; (3) subtraction of whole numbers; and (4) division of whole numbers. The instrument comprised 20 items in a short-answer (fill-in-the-blank) format, distributed equally across four arithmetic domains: addition, subtraction, multiplication, and division (5 items each), as presented in Table X. The instrument was developed by Yohanes Surya [12], [13], the originator of the GASING method, ensuring alignment with the specific arithmetic competencies targeted by the training program.

The training intervention consisted of an intensive 10-day (80-hour, 8 hours per day) GASING method program, structured in accordance with the concrete-abstract-mental arithmetic progression advocated by Surya [12], [13]. Each session commenced with ice-breaking activities and GASING songs to establish an enjoyable learning atmosphere, reflecting the method's core principle of making mathematics *gampang, asyik, dan menyenangkan* (easy, fun, and enjoyable) [12]. The detailed training schedule is presented in Table 1.

Table 1. GASING Method Training Schedule (West Manggarai Regency, October 2025)

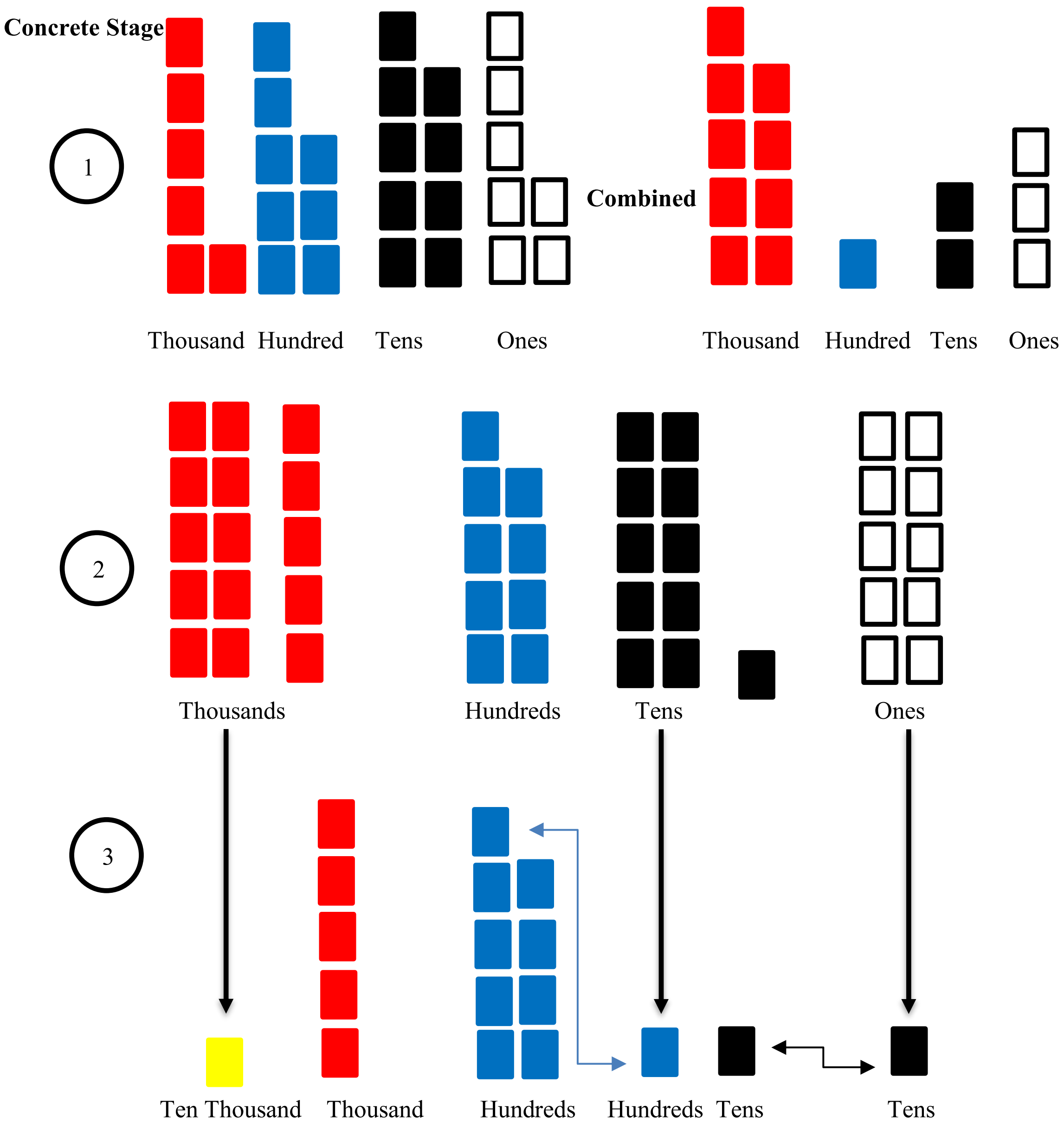
Day	Date	Topics and Activities
1	13 Oct 2025	Opening ceremony; pretest administration; introduction to GASING philosophy; addition of numbers 1–10 (results ≤10) using concrete manipulatives (fingers, straws, ice cream sticks, place value cards)
2	14 Oct 2025	Addition of numbers 11–19 (place value introduction, tens and units); addition strategies: 10+, 9+, 8+, 7+, 6+; addition of 2-digit numbers
3	15 Oct 2025	Addition of 3-digit numbers (concrete → abstract → mental arithmetic); addition of multiple numbers; introduction of large numbers (100–999,999,999)
4	16 Oct 2025	Addition of multi-digit numbers; word problems (addition); introduction to multiplication concept; multiplication by 1, 9, 10, 2, 5, and squares 1–10
5	17 Oct 2025	Multiplication by 3, 4, 8, 7, 6; review of multiplication 1–10; multiplication of 2-digit × 1-digit and 2-digit × 2-digit numbers (with and without regrouping)
6	18 Oct 2025	Multiplication of 3-digit numbers; abstract practice; special multiplication patterns; squares 1–60
7	20 Oct 2025	Multi-digit multiplication; word problems; introduction to subtraction concept; subtraction below 10; Number Bonds to 10 (Pasangan 10); subtraction of teens numbers
8	21 Oct 2025	Subtraction of 2-digit and 3-digit numbers (with and without regrouping); subtraction of multi-digit numbers using alternative approach; abstract practice and word problems
9	22 Oct 2025	Division concept; division of numbers ≤10 and ≤100 (exact and with remainder); division of multi-digit numbers by 1-digit and 2-digit divisors; mixed word problems
10	23 Oct 2025	Review of all operations; posttest administration; closing ceremony

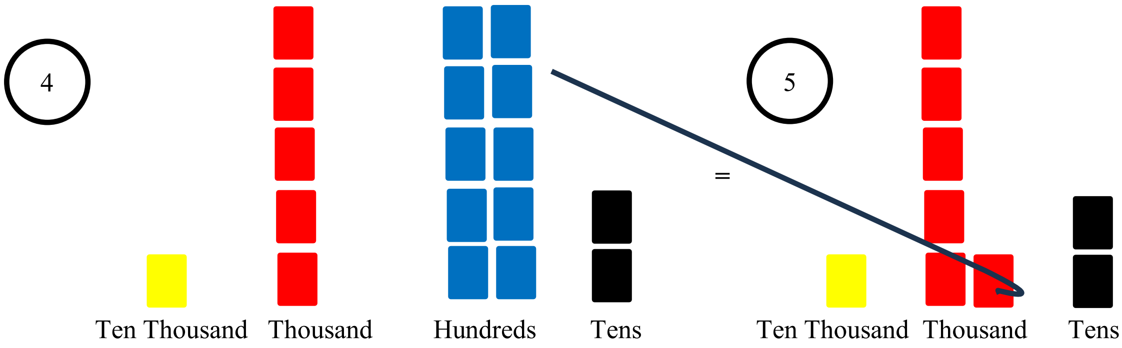
As shown in Table 1, the training followed a systematic chronological sequence aligned with the GASING critical points framework [12], [13]. Days 1–3 focused on addition, progressing from concrete manipulatives to abstract representation and mental arithmetic, consistent with the CRA framework [15], [16]. Days 4–6 explored multiplication using various concrete materials, supplemented by multiplication songs and special pattern exploration. Days 7–8 deepened understanding of subtraction by emphasizing its relationship with addition, particularly the Pasangan 10 (Number Bonds to 10) concept [12]. Day 9 explored division by integrating previously acquired multiplication and subtraction competencies. Day 10 encompassed review, posttest administration, and closing ceremony.

To illustrate the concrete-abstract-mental arithmetic progression characteristic of the GASING method, four representative examples of the instructional content delivered during the training are presented below.

Example 1: Addition

The GASING method to addition centers on the mastery of addition with results less than 20 as the foundational critical point for mental arithmetic fluency, with the place value system serving as the structural framework for multi-digit addition [12]. This three-stage progression is illustrated through the representative example of 6.897 + 9.123.





In the concrete stage, teachers employ a color-coded card system as the primary manipulative, as illustrated in Figure above. Yellow cards represent ten-thousands, red cards represent thousands, blue cards represent hundreds, black cards represent tens, and white cards represent ones. This card system operates according to a place value exchange principle: 10 ones cards are equivalent to 1 tens card, 10 tens cards are equivalent to 1 hundreds card, 10 hundreds cards are equivalent to 1 thousands card, and 10 thousands cards are equivalent to 1 ten-thousands card. For the addition 6,897 + 9,123 participants process each place value column sequentially from left to right, beginning with the largest place value. In the first step, 6 red cards (thousands) are combined with 9 red cards, yielding 15 red cards. In the second step, 8 blue cards (hundreds) are combined with 1 blue card, yielding 9 blue cards. In the third step, 9 black cards (tens) are combined with 2 black cards, yielding 11 black cards. In the fourth and final step, 7 white cards (ones) are combined with 3 white cards, yielding 10 white cards producing the final answer of 16,020.

In the abstract stage, participants record the addition process symbolically, working from left to right across each place value column. Each line represents the partial result of one place value column, written as a superscript or intermediate value alongside the running notation. The complete symbolic process for 6,897 + 9,123 is as follows:

6,897 + 9,123 =
6,897 + 9,123 = 15 → 6 thousand + 9 thousand = 15 thousand = 1 ten thousand and 5 thousand
6,897 + 9,123 = 9 → 8 hundreds + 1 hundred = 9 hundreds
6,897 + 9,123 = 11 → 9 tens + 2 tens = 11 tens = 1 hundred and 1 tens
6,897 + 9,123 = 10 → 7 ones + 3 ones =10 ones = 1 tens

How to write 6.897 + 9.123 = $159^1 1^1 0$ → 159 is written in the same capital while 1 hundreds is written in superscript letters near 9 hundreds because its place value is already available, 1 tens is written in large letters, then 1 tens is written in superscript near 1 tens and 1 ones is written in large letters.

6,897 + 9,123 = $159^1 1^1 0$

6,897 + 9,123 = $15\ 9^1\ 1^1\ 0$

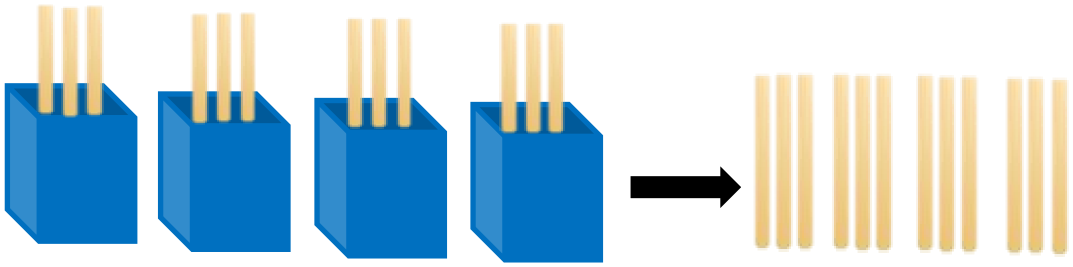
6,897 + 9,123 = $1\ 5^1\ 0\ 1^1\ 0$

6,897 + 9,123 = 16,020
Therefore, 6,897 + 9,123 = 16,020

In the mental arithmetic (*mencongak*) stage, participants perform the entire calculation mentally from left to right, processing each place value column in sequence and automatically managing carry digits, to instantly retrieve the final answer of 16,020. This structured left to right progression from the largest to the smallest place value distinguishes the GASING method from conventional right-to-left algorithms and develops teachers capacity to perform and explain multi-digit addition with both conceptual clarity and computational efficiency [12], [15], [16]. For the mental arithmetic, sthe steps are as follows. Pay attention to the number of numbers, (both consist of 4 numbers), we start counting from the front with the largest place value. 6 + 9 = 15 (we haven't mentioned 15 because there is a possibility that the result will be 15 or 16), we look back 8 + 1 = 9 (When we meet the number 9, we have to look back again because the possible result will be 9 or 10), we look back 9 + 2 = 11 that means the result in front we immediately add 1 so that 159 + 1 = 160 and 1 tens. Next we look back 7 + 3 = 10 so that 1 is immediately added to 1 tens to become 2 tens and 0 units. The final result is 16,020.

Example 2: Multiplication

For multiplication, the GASING method introduces a concrete-based approach that establishes multiplication as repeated addition, illustrated through the example of 4 × 3 [12], [13]. In the concrete stage, teachers use origami paper boxes as the primary manipulative: four boxes are prepared, each containing 3 ice cream sticks. Participants combine the contents of the boxes sequentially the first box (3) combined with the second box (3) yields 6; combined with the third box (3) yields 9; and combined with the fourth box (3) yields 12. This sequential aggregation concretely demonstrates that 4 × 3 = 12. The following image explain the multiplication procces at the concrete stage.



In the abstract stage, participants record the process symbolically: 4 × 3 = 3 + 3 + 3 + 3 = 12, directly translating the concrete manipulation into mathematical notation. In the *mental arithmetic* (*mencongak*) stage, participants automatically retrieve the answer 4 × 3 = 12 without external aids. To consolidate long-term retention of multiplication by 3 facts, the GASING method employs a dedicated multiplication song accompanied by physical movements. The song recites the multiples of 3 sequentially 3, 6, 9, 12, 15, 18, 21, 24, 27 set to an engaging rhyme that reinforces numerical patterns through auditory and kinesthetic channels [12]. This multi-sensory approach ensures that multiplication facts are not merely memorized in isolation but internalized through structured, enjoyable repetition consistent with the GASING principle of making mathematics *gampang, asyik, dan menyenangkan* (easy, fun, and enjoyable) [12], [17].

One more example for multiplying 2 numbers by 2 numbers is as follows.

Example: 67 × 89 =

67
89 × ⟶ 6 tens × 8 tens = 48 hundreds = 4 thousands dan 8 hundreds
48

67
89 × ⟶ 6 tens × 9 ones = 54 tens
54

67
89 × ⟶ 7 ones × 8 puluhan = 56 puluhan

+ 110 tens = 11 hundreds dan **no more tens**

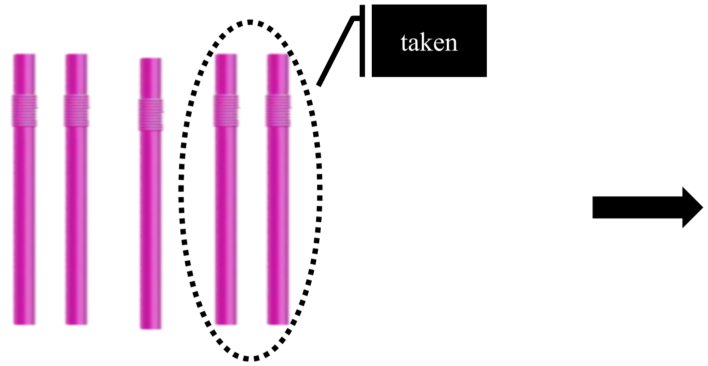
67
89 × ⟶ 7 ones × 9 ones = 63 ones = **6 tens** dan 3 ones

67
89 ×
$48^{11}\ 0^{6}\ 3 = 5{,}963$

Therefore, 67 × 89 = 5,963

Example 3: Subtraction

The GASING approach to substraction is anchored in the concept of taken, operationalized through physical manipulatives in the concrete stage: teachers use ice cream sticks, straws, pebbles, and fingers to demonstrate substraction as the physical removal od objects. For instance, beginning with 5 straws and taking away 2 yields 3 remaining straws, recorded in the abstract stage as 5-2=3. The following image explain the substraction procces at the concrete stage.

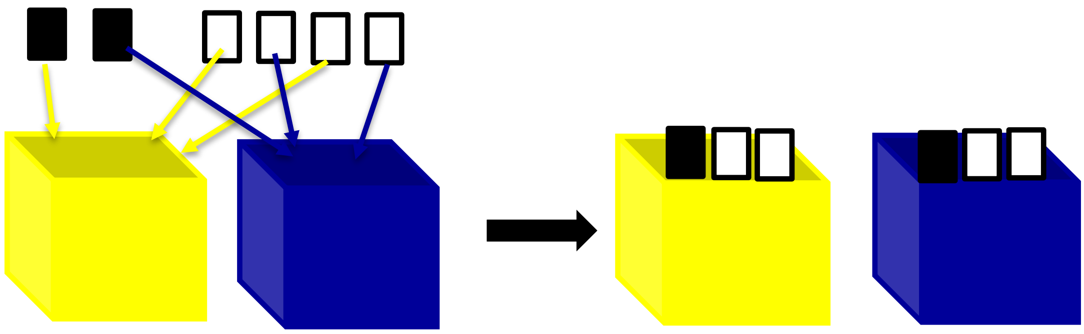


For multi digit substraction, the GASING method applies the principle of check from the back and work from the front. Participants first inspect each place value from right to left, starting from the smallest place value (ones), to determine whether the value at each place is sufficient to be substracted from and resolve any insufficiency through place value exchange by trading 1 unit of a higher place value for 10 units of the adjacent lower place value before computing from left to right. This process is illustrated through the example of 6,832 – 4,925. Upon checking from right to left: the value at the ones place (2) is unsufficient to substract 5, so 1 ten is exchanged for 10 ones, making the ones values 12 and the tens value 2; the value at the hundreds place (8) is insufficient to substract 9, so 1 thousand is exchanged for 10 hundreds, making the hundreds value 18 and the thousand value 5. The minuend is thus rewritten as $\cancel{6}^{5}.{}^{1}8\ {}^{2}\cancel{3}\ {}^{1}2 - 4.925$. The abstract stage proceeds as follows:

$\cancel{6}^{5}.{}^{1}8\ {}^{2}\cancel{3}\ {}^{1}2 - 4.925$ = check each place value from right to left, starting from the smallest place value
$\cancel{6}^{5}.{}^{1}8\ {}^{2}\cancel{3}\ {}^{1}2 - 4.925$ = 5 thousands – 4 thousands = 1 thousands
$6^{5}.{}^{1}8\ {}^{2}3\ {}^{1}2 - 4.925$ = 18 hundreds – 9 hundreds = 9 hundreds
$\cancel{6}^{5}.{}^{1}8\ {}^{2}\cancel{3}\ {}^{1}2 - 4.925$ = 2 tens – 2 tens = 0 tens
$\cancel{6}^{5}.{}^{1}8\ {}^{2}\cancel{3}\ {}^{1}2 - 4.925$ = 12 ones – 5 ones = 7 ones
Therefore, 6,832 – 4,925 = 1,907

Example 4: Division

The GASING approach to division introduces the concept through the concrete stage using boxes and the physical objects:cards, ice cream stick, straws, seeds, and pebbles are distributed equally into a specified number of boxes, and the contents of each box represent the quotient. For instance, to solve 24:2, 2 black cards (tens) and 4 white cards (ones) are distributed equally into 2 boxes, with each box receiving 1 black cards and 2 white card, yielding a quotient of 12. In the abstract stage, participants record this as 24:2 beginning with the largest palce value 2:2 = 1, then 4:2 =2, therefore 24:2 =12. The following image explain the division procces at the concrete stage.



For multi digit division, the GASING method processes each digit sequentially from left to right, carrying any remainder forward by combining it with the next digit to form a new dividend. This process is illustrated through the example of 7,059,684 : 6, as follows:

7,059,684 : 6 = 7 : 6 = 1 with a remainder of 1
$\cancel{7},{}^{1}059,684 : 6$ = 10 : 6 = 1 with a remainder of 4
$7,{}^{1}\cancel{0}\ {}^{4}59,684 : 6$ = 7 with a remainder of 3
$\cancel{7},{}^{1}\cancel{0}\ {}^{4}\cancel{5}\ {}^{3}9,684 : 6$ = 6 with a remainder of 3
$\cancel{7},{}^{1}\cancel{0}\ {}^{4}\cancel{5}\ {}^{3}9,{}^{3}684 : 6 = 6$
$\cancel{7},{}^{1}\cancel{0}\ {}^{4}\cancel{5}\ {}^{3}9,{}^{3}\cancel{6}\ 84 : 6$ = 1 with a remainder of 2
$\cancel{7},{}^{1}\cancel{0}\ {}^{4}\cancel{5}\ {}^{3}9,{}^{3}\cancel{6}\ \cancel{8}\ {}^{2}4 : 6 = 4$
$\cancel{7},{}^{1}\cancel{0}\ {}^{4}\cancel{5}\ {}^{3}9,{}^{3}\cancel{6}\ \cancel{8}\ {}^{2}\cancel{4} : 6 = 1,176,614$

Therefore, 7,059,684 ÷ 6 = 1,176,614. This left-to-right sequential processing, with remainders carried forward at each step, exemplifies the GASING method's approach to multi-digit division, enabling teachers to perform and explain division of large numbers with both conceptual clarity and computational efficiency. The four examples presented above illustrate the concrete-abstract-mental arithmetic progression that characterizes the GASING training intervention, providing the pedagogical context within which the content knowledge outcomes reported in the following section were achieved.

Data analysis employed three complementary approaches. First, the Wilcoxon signed-rank test [41] was used to examine the significance of differences between pretest and posttest scores. The selection of this non-parametric test was justified by the Shapiro-Wilk normality test results, which indicated that the posttest score distribution violated the normality assumption ($W = 0.947$, $p = 0.017$) [41]. Second, Hake's normalized gain [42] was computed using the formula: $\langle g \rangle = (S_{post} - S_{pre}) / (S_{max} - S_{pre})$, with the following interpretive criteria: $\langle g \rangle < 0.30$ (low gain), $0.30 \leq \langle g \rangle < 0.70$ (medium gain), and $\langle g \rangle \geq 0.70$ (high gain) [42]. Third, Cohen's d effect size [40] was calculated using the formula: $d = (M_{post} - M_{pre}) / SD_{pooled}$, where values of $d \geq 0.20$, ≥ 0.50, and ≥ 0.80 represent small, medium, and large effects, respectively [43]. All analyses were conducted using SPSS version 26, with a significance level set at $\alpha = 0.05$.

3. RESULTS AND DISCUSSION

3.1. Descriptive Analysis

Descriptive analysis revealed a substantial improvement in teachers' content knowledge following the GASING method training program. The mean pretest score was 31.00 (SD = 11.22; Min = 10; Max = 55), while the mean posttest score increased to 72.09 (SD = 15.08; Min = 40; Max = 95), representing an absolute improvement of 41.09 points or a relative increase of 132.55% from the baseline. The wider distribution of posttest scores indicates individual variability in response to the training intervention.

Table 2. Frequency Distribution of Pretest and Posttest Scores (N=55)

Score Range	Pretest	Posttest
0-22	8 (14.5%)	0 (0%)
21-40	32 (58.2%)	2 (3.6%)
41-60	15 (27.3%)	9 (16.4%)
61-80	0 (0%)	28 (50.9%)
81-100	0 (0%)	16 (29.1%)
Total	55 (100%)	55 (100%)

Table 2 presents the frequency distribution of pretest and posttest scores. As shown in Table 2, no teacher achieved a score above 60 on the pretest, with the highest concentration in the 21–40 range (58.2%). In contrast, 80.0% of teachers achieved scores of 60 or above on the posttest, with 29.1% reaching the 81–100 range. Furthermore, 72.7% of teachers scored below 40 on the pretest, compared to only 3.6% on the posttest. The increase in standard deviation from 11.22 to 15.08 indicates that, while all teachers demonstrated improvement, the magnitude of gain varied across individuals, with some teachers exhibiting exceptionally large knowledge gains.

3.2 Representative Work Samples of Teachers' Aritmatic Perform

To complement the statistical findings presented in Section 3.1, this section presents four representative work samples illustrating teachers' content knowledge of basic arithmetic operations as demonstrated during the GASING method training program. The samples encompass addition, multiplication, subtraction, and division, and are analyzed with respect to the application of GASING-specific computational procedures.

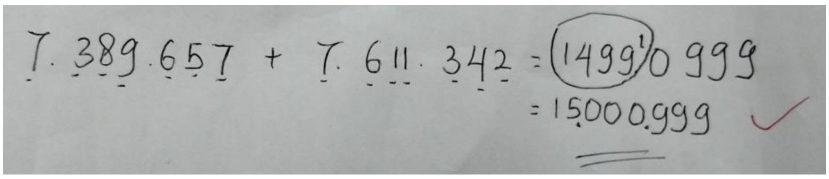


Figure 1. A representative work sample of 7-digit addition demonstrating the nine pattern in the GASING method

This problem involves the addition of 7 digit numbers and serves as an example of the nine pattern application in the GASING method. The nine pattern occurs when the digit 9 in an intermediate result turns into 0 because the sum of the digits behind it is greater than or equal to 10, causing the leading digit to increase by 1. In the work sample above, the teacher successfully added 7,389,657 and 7,611,342. The teacher first obtained

14,999,999 where 1,499 (circled) represents the sum of the leading digits, which subsequently becomes 1,500 upon application of the nine pattern, yielding the final result of 15,000,999. This demonstrates that the teacher has correctly understood and applied the concept of the nine pattern within the GASING method.

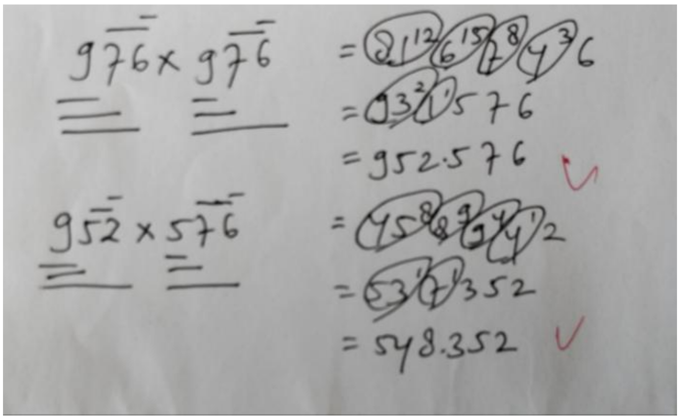


Figure 2. Representative work samples of 3-digit by 3-digit horizontal multiplication using the GASING method

The work samples above present the multiplication of two 3-digit numbers performed horizontally using the GASING method. In the first problem, the teacher correctly multiplied 976 × 976, producing the result 952,576. In the second problem, the teacher correctly multiplied 952 × 576, yielding 548,352. Both solutions demonstrate that the teacher has successfully applied the horizontal multiplication procedure in the GASING method for 3-digit by 3-digit numbers.

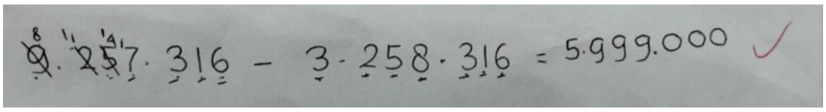


Figure 3. A representative work sample of 7-digit subtraction demonstrating the nine pattern in the GASING method

The work sample above presents a 7-digit subtraction problem and serves as an example of the nine pattern application in the GASING method. The teacher successfully solved 9,257,316 − 3,258,316, yielding the correct result of 5,999,000. As evident in the teacher's work, the subtraction process was carried out by exchanging values from the preceding digit indicated by the crossed-out numbers and small digits written above resulting in a sequence of nines in the final answer. The appearance of consecutive nines in the subtraction result is a hallmark of the nine pattern in the GASING method, demonstrating that the teacher has correctly understood and applied the concept of exchanging in the context of large-number subtraction operations.

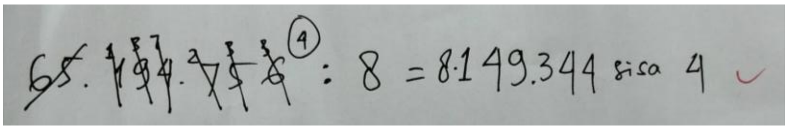


Figure 4. A representative work sample of 8-digit by 1-digit division using the GASING method

The work sample above presents a division problem of an 8-digit number by a 1-digit number, solved using the GASING method. The teacher successfully computed 65,194,756 ÷ 8, yielding the correct result of 8,149,344 remainder 4. As evident in the teacher's work, the division process was carried out horizontally, digit by digit from left to right, whereby the remainder from each division step is recorded as a small superscript above the subsequent digit as indicated by the small number (4) written above the dividend. This process reflects the characteristic approach of the GASING method in division operations, in which each digit is processed sequentially, thereby fostering both computational speed and arithmetic accuracy among the teachers.

The four representative work samples presented in Figures 1–4 collectively demonstrate that, following the GASING method training, teachers were able to correctly apply GASING-specific computational procedures including the nine pattern, horizontal multiplication, place value exchange, and sequential digit-by-digit division across all four basic arithmetic operations. These qualitative findings corroborate the statistically significant improvement in content knowledge reported in the quantitative analyses presented in the subsequent sections.

3.4 Inferential Analysis

Prior to inferential analysis, the Shapiro-Wilk normality test was conducted. Results indicated that the pretest score distribution satisfied the normality assumption ($W = 0.961$, $p = 0.068$); however, the posttest score distribution significantly violated normality ($W = 0.947$, $p = 0.017$). Given this violation, the non-parametric Wilcoxon signed-rank test [38] was deemed appropriate for comparing pretest and posttest scores.

The Wilcoxon signed-rank test revealed a highly significant difference between pretest and posttest scores ($Z = -6.441$, $p < 0.001$). All 55 teachers demonstrated score improvements (positive ranks = 55), with no teachers exhibiting score decreases (negative ranks = 0) or unchanged scores (ties = 0). These findings provide strong evidence that the GASING method training consistently and universally improved teachers' content knowledge of basic arithmetic operations.

Normalized Gain and Effect Size

Hake's normalized gain analysis [42] yielded a mean $\langle g \rangle$ of 0.5877 (SD = 0.17), classified within the medium gain category ($0.30 \leq \langle g \rangle < 0.70$). Analysis of individual N-gain distributions revealed that 2 teachers (3.6%) achieved low gain ($\langle g \rangle < 0.30$), 41 teachers (74.5%) achieved medium gain, and 12 teachers (21.8%) achieved high gain ($\langle g \rangle \geq 0.70$). Consequently, 96.4% of teachers attained at least a medium level of improvement, demonstrating a high degree of consistency in training outcomes across participants.

To further ilustrate the individual variation in training outcomes, Table 3 present the pretest scores, posstest scores, N-gain values, and qualitative comments for three representative teachers selected to reflect low, medium, and high improvement trajectories across the 55 participants.

Table 3. Individual Score Profiles of Three Representative Teachers

Teacher	Pretest	Posstest	N-Gain	Comment
Teacher A (G2)	60	70	0.25 (Low)	Although Teacher A entered training with a relatively high baseline score, the posttest improvement of 10 points reflects a ceiling-constrained gain consistent with reduced learning potential among higher-performing participants.
Teacher B (G53)	35	70	0.54 (Medium)	Teacher B represents the typical training participant, progressing from a low pretest score to the minimum mastery threshold, with an N-gain closely approximating the class mean of 0.5877.
Teacher C (G36)	70	100	1.00 (High)	Teacher C achieved a perfect N-gain of 1.00, attaining the maximum possible score of 100 on the posttest. This outcome demonstrates that even participants with higher baseline knowledge can achieve complete mastery through the structured GASING training model.

As shown in Table 3, the three teachers demonstrate markedly different improvement trajectories that collectively illuminate the diversity of responses to the GASING training intervention. Teacher A (G2) exhibited a low N-gain of 0.25, attributable primarily to a ceiling effect: entering training with a relatively high pretest score of 60 left limited room for improvement, a phenomenon consistent with findings in intensive teacher training research [23]. Teacher B (G53) exemplifies the typical participant profile, achieving an N-gain of 0.54 that closely mirrors the class mean of 0.5877, progressing from a low baseline of 35 to a posttest score of 70. Teacher C (G36) achieved a perfect N-gain of 1.00, improving from 70 to the maximum score of 100, demonstrating that the GASING training model is sufficiently rigorous and comprehensive to elevate even higher-performing participants to complete mastery. Collectively, these individual profiles corroborate the quantitative finding that the training was effective across all ability levels, with 100% of teachers demonstrating positive gains regardless of initial knowledge level.

Cohen's d effect size [40] was 2.94, substantially exceeding the threshold for a very large effect ($d > 0.80$) [43]. This value indicates that the mean posttest score was approximately three standard deviations above the mean pretest score, reflecting an extraordinarily substantial change in teachers' content knowledge [44]. This effect size is markedly larger than those reported in related teacher-training research, for instance, Sitabkhan et al. [45] reported effect sizes of 0.4–0.8 for elementary teachers in developing-country contexts, while Woods and Copur-Gencturk [25] reported an effect size of 0.6 for PCK development after two years of teaching experience. However, these comparisons should be interpreted with caution, as the studies differ considerably in intervention duration, participant characteristics, and measurement instruments; direct numerical comparison of effect sizes across such heterogeneous designs may therefore overstate the relative magnitude of GASING's impact.

The findings of this study provide robust empirical evidence of the effectiveness of GASING method training in improving elementary school teachers' content knowledge. The 132.55% improvement substantially exceeds gains typically reported in mathematics teacher professional development literature, which generally range from 30–60% [33]. Atawolo and Hartoyo [46] found that professional development programs for Indonesian elementary teachers focusing simultaneously on content knowledge and classroom practice produced significant competency gains, consistent with the present findings. This remarkable difference in magnitude can be attributed to several distinctive pedagogical mechanisms of the GASING training model, which are discussed in the following sub-sections.

First, the GASING approach prioritizes conceptual understanding prior to procedural mastery, directly aligned with the principles of mathematical knowledge for teaching as defined by Ball et al. [1]. Through GASING training, teachers did not merely learn how to solve arithmetic operations, but also why specific methods work, how mathematical concepts are interrelated, and how to represent mathematical ideas through multiple modalities, contall of which constitute essential competencies for effective instruction. Rowland et al. [47] emphasized that knowledge for teaching elementary mathematics encompasses both content and contingency dimensions that are mutually reinforcing, while Li et al. [48] underscored that mathematical proficiency for elementary instruction requires deep understanding that transcends procedural knowledge. Hill et al. [2], [3] demonstrated that the depth of teachers' conceptual understanding directly correlates with their capacity to anticipate student difficulties and select appropriate representations, further validating the pedagogical value of the GASING conceptual-first approach.

Second, the present findings corroborate the well-established effectiveness of concrete manipulatives noted in Section 1 [17]; the training's substantial gains suggest that GASING's structured use of manipulatives facilitated the construction of robust mental schemas among teachers, consistent with the concrete-to-abstract progression documented by Huan et al. [15], through a bibliometric analysis spanning over five decades (1968–2021), confirmed that the concrete-to-abstract learning progression has consistently emerged as the dominant and effective approach in mathematics education research. Milton et al. [16] specifically demonstrated that the concrete-representational-abstract sequence significantly enhances conceptual understanding of multiplication and division operations. Critically, within the context of teacher professional development, this approach not only strengthens teachers' content knowledge but simultaneously models effective pedagogy that teachers can directly transfer to their own classroom practice [49], thereby generating a dual benefit of the GASING training program. Third, the training's 10-day (80-hour) duration appears to have provided the sustained engagement that Kennedy [33] and Desimone [34], [35] identify as prerequisite for deep conceptual internalization, rather than superficial exposure. The near-universal improvement observed across all 55 participants, regardless of prior competency, lends empirical support to the claim by Darling-Hammond et al. [36] and Garet et al. [37] that duration and hands-on intensity jointly determine training impact.

The remarkable consistency of training outcomes with 96.4% of teachers achieving at least medium N-gain stands in notable contrast to the variability reported by Hassler et al. [28], in which only 60–70% of teachers in regions with limited access demonstrated substantial improvement through conventional training programs. This finding suggests that the instructional design of GASING characterized by a structured learning trajectory progressing from concrete to abstract mental arithmetic effectively accommodates the diversity of teachers' backgrounds and initial knowledge levels. This adaptability is consistent with Van de Walle et al. [51], who emphasized the critical importance of systematic scaffolding in mathematics instruction, enabling learners at varying proficiency levels to construct meaningful understanding. Furthermore, the universality of improvement across all 55 participants regardless of their baseline knowledge highlights the inclusive and accessible nature of the GASING training model, a particularly significant characteristic given the heterogeneous professional backgrounds typical of teachers in regions with limited access to quality professional development.

The Cohen's d effect size of 2.94 positions the GASING training program among the most impactful interventions reported in the mathematics teacher professional development literature. Kleickmann et al. [52] demonstrated that structural differences in teacher training programs significantly influence both content knowledge and pedagogical content knowledge acquisition, underscoring the importance of well-designed training architecture such as that of GASING. Fukaya et al. [26] found that strong PCK correlates positively with teaching motivation, suggesting that the content knowledge gains produced by GASING training may simultaneously reinforce teachers' professional commitment and instructional confidence. In this regard, Bandura [53] established that self-efficacy constitutes a powerful predictor of behavioral change; accordingly, the content mastery acquired through GASING training may substantially strengthen teachers' teaching self-efficacy. Zee and Koomen [54] further demonstrated that high teacher self-efficacy positively influences classroom processes and teacher well-being, indicating that the benefits of GASING training extend well beyond the cognitive domain. Collectively, Fauth et al. [55] and Hanushek et al. [56] converge in demonstrating that strong teacher content knowledge, rather than merely procedural competency, mediates student learning outcomes and cognitive performance, suggesting that investment in GASING-style training is likely to yield substantial downstream benefits for student achievement. Arifin et al. [57] reported large effect sizes of Cohen's d = 0.95 and d = 1.43 for character maturity and 21st-century competencies, respectively, in a quasi-experimental STEAM-integrative curriculum intervention (N = 200). While the effect size observed in the present study (d = 2.94) numerically exceeds these values, such comparisons warrant caution given the differing outcome constructs, intervention durations, and measurement approaches; the present findings are therefore best interpreted as evidence of a substantial within-sample effect rather than a definitive ranking of intervention effectiveness relative to prior studies.

The geographical context of West Manggarai Regency further amplifies the significance of these findings. The World Bank [8], [39] has documented that geographic isolation and infrastructure limitations constitute primary barriers to educational quality in Indonesia's peripheral regions, restricting teachers' access to sustained professional development. Rowan et al. [58] demonstrated that teacher effects on student learning outcomes are substantial and frequently exceed the influence of other school-level factors; consequently, improving teacher content knowledge through GASING training has the potential to generate cascading positive effects on the quality of mathematics instruction across West Manggarai schools. Dietrichson et al. [59], through a systematic review, found that structured content-based interventions produce significant and consistent impacts on mathematics achievement, corroborating the progressive and structured nature of the GASING approach. In this context, the GASING training model offers a feasible and highly effective alternative, as it can be implemented with minimal resources, utilizing locally available, low-cost manipulatives while yielding maximum impact. This stands in sharp contrast to technology-dependent professional development programs, which are frequently unsustainable in regions with limited infrastructure [8], [28].

Nevertheless, several limitations of this study warrant acknowledgement. First, the pre-experimental design without a control group [38] constrains causal inference; the potential influence of testing effects or the Hawthorne effect cannot be entirely precluded. Future research employing quasi-experimental or experimental designs with control groups would provide stronger causal evidence for the effectiveness of GASING training. Second, measurement was restricted to teachers' content knowledge and did not extend to actual classroom practice or student learning outcomes; subsequent studies should therefore investigate the translation of content knowledge gains into pedagogical practice [2], [3]. Third, as the posttest was administered immediately following the training program, the long-term retention of acquired knowledge remains unexamined [30]. Christiansen and Erixon [60] emphasized the critical role of reflective practice in consolidating PCK gained through formal training, suggesting that follow-up support mechanisms such as peer coaching, lesson study, or professional learning communities at the regency level may be essential for sustaining and deepening the impact of GASING training over time. In this context, Kurniawan et al. [61] established that rigorous curriculum construction significantly predicts professional readiness ($\beta = 0.467$, $p < 0.001$) and that soft skills proficiency serves as the critical mediating bridge between curriculum architecture and professional preparedness suggesting that future GASING training designs should systematically embed transferable professional competencies to maximize downstream impact beyond the training period.

4. CONCLUSION

This study demonstrated that the GASING method training program significantly improved elementary school teachers' content knowledge of basic arithmetic operations in West Manggarai Regency, East Nusa Tenggara, Indonesia. The three research objectives stated in the introduction were fully addressed: (1) teachers' mean content knowledge score increased substantially from 31.00 to 72.09, representing a 132.55% improvement; (2) the normalized gain of 0.5877 placed the overall outcome in Hake's medium category, with 96.4% of teachers achieving at least medium-level improvement; and (3) Cohen's d of 2.94 confirmed a very large effect size, indicating that the training produced an extraordinarily impactful and consistent change in teachers' content knowledge. These findings collectively establish that the GASING training model with its structured concrete-abstract-mental arithmetic progression, intensive 10-day format, and low-resource manipulative based design constitutes an effective, replicable, and scalable professional development approach for elementary mathematics teachers, particularly in regions with limited access to conventional training programs. From the perspective of future development, several promising research directions emerge from these findings. First, longitudinal studies are needed to examine the retention of content knowledge gains and their translation into sustained improvements in classroom practice and student learning outcomes. Second, quasi-experimental designs incorporating control groups would strengthen causal claims regarding the effectiveness of GASING training. Third, the GASING training model warrants investigation across diverse geographical and cultural contexts within Indonesia and other developing countries, to examine its generalizability and scalability as a low-resource professional development model. Fourth, future studies could explore the integration of digital tools with GASING manipulatives to extend the reach of the training program to teachers in the most geographically isolated communities. Finally, the development of teacher learning communities following GASING training could be examined as a mechanism for sustaining and amplifying professional growth beyond the initial training period.

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AUTHOR CONTRIBUTIONS

Conceptualization, Y.N.B and T.; Methodology, Y.N.B.; Software, Y.N.B.; Validation, Y.N.B., T., and E.M.; Formal Analysis, Y.N.B.; Investigation, Y.N.B.; Data Curation, Y.N.B.; Writing – Original Draft Preparation, Y.N.B.; Writing – Review & Editing, T., E.M., and E.P.; Visualization, Y.N.B.; Supervision, T. and E.M.; Project Administration, Y.N.B.

CONFLICTS OF INTEREST

The author(s) declare no conflict of interest.

INFORMED CONSENT STATEMENT

Informed consent was obtained verbally from all subjects involved in the study. Prior to participation, each subject was provided with a detailed explanation of the study's objectives, procedures, potential risks, and benefits. All participants voluntarily agreed to participate in the GASING method training program. All images in this article have been authorized and are the property of the authors.

USE OF ARTIFICIAL INTELLIGENCE (AI)-ASSISTED TECHNOLOGY

The authors declare that no artificial intelligence (AI) tools were used in the generation, analysis, or writing of this manuscript. All aspects of the research, including data collection, interpretation, and manuscript preparation, were carried out entirely by the authors without the assistance of AI-based technologies.

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