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# Thinking Process of Mathematics Education Students in Problem Solving Proof

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**ABSTRACT**

**Purpose of the study:** The purpose of this study is the thinking process of mathematics education students in solving proving problems.

**Methodology:** This type of research is quantitative research. The sample technique used is purposive sampling technique. The subjects in this study were 6 semester 2 students of the Mathematics Education study program. This study used document analysis, interviews and evidence problem solving task instruments. Qualitative data analysis was carried out interactively.

**Main Findings:** The results of this study are the thinking processes of 2nd semester Mathematics Education students at Nusantara PGRI Kediri University who have high learning achievements. Solving the problem of proof in a direct way, contraposition, and contradiction in the entry phase of the thought process activity obtained is the same, that is, the subject understands the problem by writing down the antecedents as what is known and the consequent as what must be proven. The thinking process of 2nd semester Mathematics Education students at Nusantara PGRI Kediri University who have moderate learning achievements. Solving the problem of proof in a direct way, contraposition, and contradiction in the entry phase of the thought process activity obtained is the same, that is, the subject understands the problem by writing down the antecedents as what is known and the consequence as what must be proven. The thinking process of 2nd semester Mathematics Education students at Nusantara PGRI Kediri University who have low learning achievements. Solving the problem of proof in a direct way, contraposition, and contradiction in the entering phase of the thinking process activity obtained is the same, that is, the subject understands the problem by writing down the antecedents as what is known and the consequent as what must be proven.

**Novelty/Originality of this study:** Dapat mengetahui sejauh mana proses berpikir mahasiswa dalam pemecahan masalah pembuktian dan sebagai bahan pembelajaran untuk lebih mengetahui argumentasi pembuktian yang dihasilkan mahasiswa sehingga dapat digunakan sebagai evaluasi untuk pembelajaran selanjutnya.



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## 1. INTRODUCTION

Education for mankind is an absolute necessity that must be met throughout life. Education is the basic capital for improving the quality of human resources so that humans are required to continue to learn, understand, and master various kinds of scientific disciplines to then be applied in all aspects of life [1]–[3]. Mathematics has a very important role in solving problems in everyday life, such as calculating, processing and presenting data, using calculators and computers, and so on. Mathematics trains students' ways of thinking and reasoning in drawing conclusions, which is carried out through investigation, exploration, experimentation, showing similarities or differences and so on. In addition, the main task of mathematics education is to explain students' thinking processes in learning mathematics with the aim of improving mathematics learning in schools [4], [5].

Thinking is a mental activity experienced by someone when they are faced with a problem or situation that must be solved. Thinking occurs in every person's mental activity, which functions to formulate or solve problems, make decisions, and seek understanding of something [6]–[9]. The thought process is needed by everyone in carrying out activities of daily life, namely to be able to survive in conditions that are always changing, uncertain and competitive as they are today. Everyone does a thought process to solve a problem. Likewise in mathematics, to be able to solve mathematical problems must go through a thought process so that in the end a solution or solution can be obtained. Problem solving is defined as a more complex process than citing numbers from problems to solve an equation [10], [11]. Problem solving is a process of thinking, testing hypotheses and finding solutions so that something new can be learned from this process.

Mathematical problem solving is considered the essence of mathematics because various kinds of mathematical activities are to solve problems. Problem solving ability is a very important part of learning mathematics [12]. Problem solving ability is a process that prioritizes the importance of a step taken by students in solving problems and finally being able to find answers to questions [13], [14].

Mathematics learning is often assessed negatively by students and they have considerable difficulties with some mathematical processes such as reasoning, non-routine problem solving, and proof [13], [15]. This difficulty is a problem for students caused by a change from elementary thinking to advanced mathematical thinking, namely from describing to defining and from convincing to logically proving based on a definition. In solving a proving problem it is possible that each student has different thinking abilities, because the thinking and analysis processes of each student are different.

The purpose of this study is the thinking process of mathematics education students in solving proving problems. The questions posed in this study are as follows.

1. What is the thinking process of mathematics education students who have high learning achievements in solving direct proof problems, contrapositions, and contradictions?
2. What is the thinking process of mathematics education students who excel in learning while solving problems of direct proof, contraposition, and contradiction?
3. What is the thinking process of mathematics education students with low learning achievement in solving direct proof problems, contrapositions, and contradictions?

## 2. RESEARCH METHOD

This research is a qualitative research. Qualitative research is a research procedure that produces descriptive data in the form of written or spoken words from people and observable behavior [16]–[18]. In this study, we will observe the thinking process in solving proving problems for mathematics education students who have high, medium, and low learning achievements. Determination of the subject by using purposive sampling. Purposive sampling is a sampling technique with certain considerations [19]. The subjects in this study were 6 second semester students. This study used document analysis, interviews and evidence problem solving task instruments. Qualitative data analysis is carried out interactively and continues continuously until complete so that the data is saturated, while the stages are data collection, data reduction, verification and conclusions [20], [21].

## 3. RESULTS AND DISCUSSION

From the results of data analysis of students' thinking processes in solving evidentiary problems based on three phases of the thinking process, namely the initial phase, the finishing phase, and the review phase. In each phase, it will be seen how the thinking processes carried out by students in solving the problem of proof directly and indirectly in a way of contraposition and contradiction. The following will explain the thought process in solving proof problems for each level of student achievement with high, medium, and low learning achievements.

### 3.1. The thinking process of students who have high learning achievements

#### 3.1.1 The thinking process of students on the AA subject and MY subject on solving the problem of proof directly

The similarities between AA subject and MY subject in solving direct proof problems are as follows. In the entry phase, both subjects can understand the problems given. The subject writes down what is known from the problem correctly, namely information on the question and antecedents. Next, the subject writes down what must be proven from the problem correctly, namely consequent. In the completion phase, the two subjects complete the proof that the sum of two odd numbers is an even number correctly. Next, the subject performs arithmetic operations by adding two numbers and carrying out the distributive property to link the consequent to the premise. The subject can complete the proof process correctly and systematically both in terms of algebra and the conditions for even numbers. The subject also shows the consequences correctly and completely with the conditions that meet them. In the second review phase, the subject examines the process of proof and believes that the solution has been able to prove the problems that have been proven.

The difference is that subject AA in writing examples for two odd numbers is considered the same, while subject MY writes examples for two different odd numbers. Differences in thought processes that occur in the form of differences in stating the problem. This agrees with Montague (in Zhu, 2007: 188) which means, in solving problems there are two steps, namely stating the problem and solving the problem. Both are considered to represent the problem as a basis for understanding the problem and making plans to solve the problem. From this opinion, the activity or process of compiling and implementing a plan based on the knowledge obtained from each subject to solve a problem.

#### 3.1.2 The process of thinking in solving indirect proof problems in contraposition

The similarities between the subject AA and subject MY in solving the problem of indirect proof in contraposition are as follows. In the initial phase the two subjects understood the problem given. The subject also wrote down what was known from the problem, namely the information contained in the problem. Next, the subject writes down what must be proven from the question, namely the implication statement that will be proven. After that, the subject forms a contrapository statement which will be proven correctly, namely by contraposing known implication statements. In the solving phase, the two subjects relate the consequent to the premise that has been obtained by using arithmetic operations and the nature of numbers so as to form a proof process. The proving process carried out by the subject is clear and systematic, that is, the subject can correctly exemplify the antecedents and consequences. The subject can determine the negation of the antecedent and consequent statements. The subject can also write the statement in the form of a contraposition correctly. The subject can complete the statement in algebraic form. Furthermore, it can draw conclusions about the truth of the statement as a result of the equivalence of the contrapositive statement correctly. In the second review phase, the subject re-examines the solution and the subject feels confident that the solution has been able to prove the problems that have been proven. In AA and MY subjects there is no difference in solving indirect proof problems by means of contraposition.

#### 3.1.3 The process of thinking in solving indirect proof problems by contradiction

The similarities between the subjects AA and MY in solving the problem of indirect evidence by contradiction are as follows. In the initial phase, the subject can understand how to prove the contradiction and the subject can also determine what will be proven, namely the contradiction (denial) is false. In the completion phase, the two subjects can form the statement in the form of an implication statement. The subject can determine the circle of statements with implications. Subjects can form statements with conjunctions that are commutative. The subject can complete the statement in algebraic form. The subject can determine the circle results are wrong. Furthermore, the subject can draw conclusions about the truth of the statement, namely the contradiction is false. In the second review phase, the subject re-examines the solution and the subject feels confident that the solution has been able to prove the problems that have been proven. In AA and MY subjects there is no difference in solving indirect proof problems by means of contradiction.

### 3.2. The thinking process of students who have moderate learning achievements

#### 3.2.1 The thinking process of students on the subject of PAD and the subject of EF is on solving direct proof problems

The similarities between PAD subjects and EF subjects in solving direct proof problems are as follows. In the initial phase of the thinking process that was carried out by the two subjects was to understand the problem by writing down what was known about the problem and writing down what had to be proven from the problem correctly, namely consequent. In the phase of completing the thinking process carried out by the two subjects, the subject completed proving the sum of two odd numbers is an even number correctly. Next, the subject performs arithmetic operations by adding two numbers and carrying out the distributive property to link the consequent to the premise. The subject can complete the proof process correctly and systematically both in

terms of algebra and the conditions for even numbers. The subject also shows the consequences correctly and completely with the conditions that meet them. In the second review phase the subject did not check the answers that had been made. Furthermore, the subject feels confident that the solution has been able to prove the problems that have been proven.

The difference is in the finishing phase for the PAD subject explaining and completing correctly with only one explanation, while the EF subject explains and completes the steps of proof with repeated explanations until it reaches the completion of the proof truth. This is supported by Hanna's opinion (in Imamoglu and Togrol, 2010:79) which means, mathematical proof is considered as an argument needed to legitimize a statement, an argument that may take several different forms or ways as long as it is convincing. From this opinion, the proving process carried out by the two subjects has a different way of completing the truth of the proof.

### 3.2.2 The process of thinking in solving indirect proof problems in contraposition

The similarities between the PAD subject and the EF subject in solving the problem of indirect proof by contraposition are as follows. In the second entry phase the subject understands the problem given. The subject writes down what is known from the problem, namely the information contained in the problem. Next, the subject writes down what must be proven from the question, namely the implication statement that will be proven. After that, the subject forms a contrapository statement which will be proven correctly, namely by contraposing known implication statements. In the final phase of the thinking process, the subject describes the antecedents into clearly related premises. Furthermore, the subject relates the consequent to the premise that has been obtained by using arithmetic operations and the nature of numbers so as to form a proof process. The proving process carried out by the subject is clear and systematic, that is, the subject can correctly exemplify the antecedents and consequences. The subject can determine the negation of the antecedent and consequent statements. The subject can also write the statement in the form of a contraposition correctly. The subject can complete the statement in algebraic form. Furthermore, it can draw conclusions about the truth of the statement as a result of the equivalence of the contrapositive statement correctly. In the second review phase, the subject does not re-examine the solution and the subject feels confident that the solution has been able to prove the problem that has been proven. In the PAD and EF subjects, there is no difference in solving the problem of indirect proof by contraposition.

### 3.2.3 The process of thinking in solving indirect proof problems by contradiction

The similarity of thought processes in PAD subjects and EF subjects is as follows. In the entry phase that was carried out by the two subjects, the subject was able to write the questions correctly. The subject can understand the method to be proven with contradictory evidence and the subject can also determine what will be proven, namely the contradiction (denial) is false. The final phase is that the subject can form the statement in the form of an implication statement. The subject can determine the circle of statements with implications. Subjects can form statements with conjunctions that are commutative. The subject can complete the statement in algebraic form. The subject can determine the circle results are wrong. Furthermore, the subject can draw conclusions about the truth of the statement, namely the contradiction is false. In the review phase, the thinking process carried out by the two subjects was that the subject did not re-examine the solution and the subject felt confident that the solution had been able to prove the problems that had been proven. In PAD and EF subjects there is no difference in solving the problem of indirect proof by contraposition. While the differences in the thought processes of each subject were found in the finishing phase for PAD subjects explaining and completing correctly with only one explanation, while MY subjects explained and completed the steps of proof with repeated explanations until they reached the completion of the proof truth. This is supported by Hanna (in Imamoglu and Togrol, 2010:79) which means, mathematical proof is considered as an argument needed to validate a statement, an argument that may take different ways as long as it is convincing. From this opinion, the proving process carried out by the two subjects has a different way of completing the truth of the proof.

## 3.3 The thinking process of students with low learning achievements

The thinking process of students on the subject of YNK and DW in solving direct proof problems. The similarity of the thinking process on the YNK subject and the DW subject is as follows. In the second entry phase, the subject can understand the problem by writing the problem that the sum of two odd numbers is an even number, and the two subjects understand what will be proved. The phase of completing the two subjects lacked understanding in exemplifying the antecedents. The subject is not correct in showing the consequences and the subject also does not understand the statement that must be proven. In the second review phase, the subject does not re-examine the evidence process and the subject is sure of the answer.

### 3.3.1 The thought process for solving indirect proof problems by way of contraposition

The similarity of the thought processes in the YNK subject and the DW subject are as follows. In the entry phase, the subject can write the questions correctly and the subject can understand the contrapositive

statement that will be proven. In the finalizing phase the subject is inconsistent in the examples of the statements written. Although the conclusion is correct, the process of proving it is not correct. The subject review phase does not re-examine the evidence process and the subject is sure of the answer.

### 3.3.2 The thought process for solving indirect proof problems by way of contradiction

As for the similarities of the subjects YNK and DW in the initial phase the subject wrote the questions correctly and the subject did not understand the questions to be proven because the subject did not form contradictory statements to be proven. The phase of completing the subject is incorrect in writing algebraically. The subject can conclude that the contradiction is false, but cannot explain that the statement is true. The subject cannot prove the consequence because the proof process that was carried out was not correct. The subject review phase does not re-examine the evidence process and the subject is sure of the answer. The difference in the thought process of solving indirect proof problems by contradiction found in the completion phase of the YNK subject is that the subject cannot exemplify numbers that are not divisible by 3, and the DW subject can exemplify numbers that are not divisible by 3.

## 4. CONCLUSION

Based on the results of data analysis and discussion, the following conclusions can be obtained. The thinking process of 2nd semester Mathematics Education students who have high learning achievements. Solving the problem of proof in a direct way, contraposition, and contradiction in the entry phase of the thought process activity obtained is the same, that is, the subject understands the problem by writing down the antecedents as what is known and the consequent as what must be proven. Completion phase, the subject is correct and complete in describing the antecedents into premises, performing calculation operations with addition steps and distributive properties to link the consequent with the premise, correct and clear in proving the consequent. Solving the problem of indirect proof by means of contraposition and contradiction in the completion phase, the subject is correct and complete in describing the antecedents to the premises, performs calculation operations with squaring steps and distributive properties to associate the consequences with the premises, is correct in proving the consequences. Furthermore, in the review phase for solving the problem of proof in a direct way, contraposition, and contradiction, the subject checks the answers and is sure of the answers after seeing the process and results of the proof. The thinking process of Mathematics Education students in semester 2 who have moderate learning achievements. Solving the problem of proof in a direct way, contraposition, and contradiction in the entry phase of the thought process activity obtained is the same, that is, the subject understands the problem by writing down the antecedents as what is known and the consequence as what must be proven. Completion phase, the subject is correct in describing the antecedent to the premise, performs arithmetic operations with the addition step and distributive properties to associate the consequent with the premise, is correct in proving the consequent. Solving the indirect proof problem by means of contraposition and contradiction in the settlement phase, the subject is correct in translating the antecedents into premises, performs calculation operations with squaring steps and distributive properties to link the consequences to the premises, is correct in proving the consequences. Furthermore, in the review phase for solving the problem of proof in a direct way, contraposition, and contradiction, the subject does not check the answer and is sure of the answer when the evidence is proven. The thinking process of 2nd semester Mathematics Education students who have low learning achievement. Solving the problem of proof in a direct way, contraposition, and contradiction in the entering phase of the thinking process activity obtained is the same, that is, the subject understands the problem by writing down the antecedents as what is known and the consequent as what must be proven. Completion phase, the subject has difficulty translating the antecedents into premises, performs arithmetic operations with addition steps and distributive properties to associate the consequent with the premise by using number examples, the subject proves the consequent by forming numbers to fulfill the consequent. Solving the indirect proof problem by means of contraposition and contradiction in the settlement phase, the subject is correct in translating the antecedents into premises, performs calculation operations with squaring steps and distributive properties to link the consequent to the premise, the subject proves the consequent

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