



Comparative Effectiveness of AIR (Auditory, Intellectually, Repetition) and Realistic Mathematics Education Approaches on Students' Mathematical Reasoning Ability

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ABSTRACT

Purpose of the study: This study aims to compare the effectiveness of the AIR (Auditory, Intellectually, Repetition) and Realistic Mathematics Education (RME) approaches in improving students' mathematical reasoning ability at the senior high school level.

Methodology: This study employed a quantitative quasi-experimental method with a posttest-only design, using essay-based test instruments, interviews, observations, and documentation; data were analyzed using descriptive statistics, Liliefors normality test, F-test homogeneity, and independent sample t-test with statistical calculation procedures.

Main Findings: The results showed a significant difference between the two approaches, where the AIR approach produced higher mean scores than the RME approach in students' mathematical reasoning ability. Statistical testing indicated $t_{value} > t_{table}$, confirming that AIR is more effective in enhancing students' reasoning skills compared to RME.

Novelty/Originality of this study: This study provides a direct comparative analysis between AIR and RME approaches within a single quasi-experimental framework, focusing specifically on mathematical reasoning ability and offering empirical evidence to support instructional decision-making in mathematics education.

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1. INTRODUCTION

Mathematics plays a fundamental role in developing students' logical thinking, analytical skills, and problem-solving abilities. As a structured and hierarchical discipline, mathematical concepts are interconnected and require deep understanding rather than mere memorization [1], [2]. In the learning process, students are expected to actively construct their knowledge through meaningful experiences. Therefore, instructional approaches should facilitate active engagement and cognitive development [3]. In this context, mathematical reasoning becomes an essential competency that must be developed in students.

However, in practice, students' mathematical reasoning ability is still relatively low. Many students experience difficulties in understanding concepts, performing mathematical manipulations, and drawing logical conclusions [4], [5]. Learning activities are often teacher-centered, limiting students' opportunities to actively participate in the learning process. As a result, students tend to rely on memorization instead of developing

reasoning skills [6], [7]. This condition leads to unsatisfactory learning outcomes, as indicated by the low percentage of students achieving the minimum mastery criteria.

Previous studies have shown that innovative learning approaches can improve students' higher-order thinking skills, including mathematical reasoning. The Auditory, Intellectually, Repetition (AIR) approach has been proven effective in enhancing students' critical thinking and engagement through active listening, reasoning, and reinforcement activities [8], [9], [10]. Similarly, the Realistic Mathematics Education (RME) approach emphasizes contextual learning, enabling students to construct mathematical concepts based on real-life situations. These approaches encourage student-centered learning and promote deeper conceptual understanding. However, most studies focus on the effectiveness of each approach independently rather than comparing their impacts.

Despite the existing findings, there is still limited research that directly compares the effectiveness of AIR and RME approaches in improving students' mathematical reasoning ability. Furthermore, previous studies often involve different contexts, sample characteristics, and implementation strategies, making it difficult to generalize the results [11], [12]. In many classrooms, the selection of instructional approaches is not based on empirical comparison but rather on teachers' preferences. This creates a gap between theoretical recommendations and classroom practices [13], [14]. Therefore, a comparative study is needed to determine which approach is more effective in enhancing students' reasoning ability.

The importance of this study lies in its potential to provide empirical evidence for selecting appropriate instructional approaches in mathematics learning. By identifying the more effective approach, teachers can design learning activities that better support students' reasoning development [15], [16]. Additionally, this study contributes to improving the quality of mathematics education by promoting active and meaningful learning. The findings are expected to help students become more engaged and capable of solving mathematical problems logically [17]. Thus, this research is highly relevant to current educational challenges.

This study offers novelty by directly comparing the effectiveness of AIR and RME approaches using a quasi-experimental design with a posttest-only framework. Unlike previous research, this study focuses specifically on students' mathematical reasoning ability as the main outcome variable. It also provides empirical data based on controlled classroom implementation in a real educational setting. Furthermore, the study integrates two widely recognized approaches in a single comparative framework. Therefore, this research contributes new insights into instructional strategies for improving mathematical reasoning in secondary education.

2. RESEARCH METHOD

2.1 Research Design

This study employed a quantitative approach using a quasi-experimental method to compare the effectiveness of two learning approaches, namely Auditory, Intellectually, Repetition (AIR) and Realistic Mathematics Education (RME), on students' mathematical reasoning ability [18], [19]. A quasi-experimental design was selected because it allows comparison between groups without full control of external variables. The research utilized a posttest-only control group design to measure the effect of the treatments after the learning process. The study involved two experimental classes, where one class was taught using the AIR approach and the other using the RME approach. The posttest results were considered as the primary data to determine the effectiveness of each learning approach.

2.2 Participants and Research Procedure

The research was conducted at a senior high school involving tenth-grade science students. The population consisted of 122 students distributed across four classes. The sample was selected using a simple random sampling technique, resulting in two classes as research samples. The first class was assigned as Experimental Group I and received instruction using the AIR approach, while the second class was assigned as Experimental Group II and received instruction using the RME approach. The research procedure included preparation, implementation of learning treatments, and posttest administration. The treatments were conducted during regular mathematics class sessions in accordance with the school schedule.

2.3 Data Collection and Analysis

Data were collected using several techniques, including interviews, observations, tests, and documentation. The primary instrument was a posttest consisting of essay questions designed to measure students' mathematical reasoning ability. The instrument had been tested for validity, reliability, difficulty level, and discrimination power to ensure its quality [20], [21]. Data analysis consisted of descriptive and inferential statistics. Descriptive analysis was used to determine the mean and variance of students' scores, while inferential analysis was conducted using an independent sample t-test. Prior to hypothesis testing, prerequisite tests

including normality and homogeneity tests were performed. The hypothesis was tested at a significance level of 0.05 to determine whether there was a significant difference between the two learning approaches.

Table 1. Research Design

Group	Treatment	Posttest
Experimental I	AIR Approach	O
Experimental II	RME Approach	O

Table 2. Variables of the Study

Variable Type	Description
Independent Variable	AIR and RME learning approaches
Dependent Variable	Students' mathematical reasoning ability

3. RESULTS AND DISCUSSION

3.1 Prerequisite Test Results

Before conducting hypothesis testing, prerequisite analyses were performed, including normality and homogeneity tests. The normality test aimed to determine whether the posttest data of students' mathematical reasoning ability were normally distributed. The test was conducted using the Liliefors test at a significance level of 5%. The results indicated that the calculated Lvalue for both groups was smaller than the critical Ltable. Therefore, it can be concluded that the data in both experimental groups were normally distributed.

Furthermore, a homogeneity test was conducted to examine the equality of variances between the two groups. This test used the F-test at a significance level of 5%. The analysis showed that the value of Fvalue was less than or equal to Ftable, indicating that the variances of the two groups were homogeneous. Since both assumptions were fulfilled, the analysis could be continued using the independent sample t-test. The summary of the prerequisite test results is presented in Table 1.

Table 3. Results of Normality and Homogeneity Tests

Group	Mean	SD	Lvalue	Ltable	Normality	Fvalue	Ftable	Homogeneity
Experimental I (AIR)	86.30	8.51	0.1508	0.1617	Normal	1.5286	1.8409	Homogeneous
Experimental II (PMR)	75.10	10.53	0.1144	0.1617	Normal			

3.2 Hypothesis Testing Results

After confirming that the data were normally distributed and homogeneous, hypothesis testing was conducted using an independent samples t-test. This test aimed to determine whether there was a significant difference in students' mathematical reasoning ability between those taught using the AIR approach and those taught using the PMR approach. The results showed that the calculated tvalue = 2.3012 and the critical ttable = 2.0423 at a significance level of 5%. Since tvalue > ttable, the null hypothesis (H0) was rejected and the alternative hypothesis (H1) was accepted.

This result indicates that there is a significant difference in students' mathematical reasoning ability between the two groups. The mean score of the experimental group I (AIR) was higher than that of experimental group II (PMR). Therefore, the AIR approach is considered more effective in improving students' mathematical reasoning ability on logarithmic material.

3.3 Description of Students' Mathematical Reasoning Ability

Further analysis was conducted on students' answer sheets to examine the quality of their mathematical reasoning. In the experimental group I, most students were able to present mathematical statements clearly and completely, formulate conjectures, and perform symbolic manipulation accurately. In addition, they were able to construct logical arguments and draw appropriate conclusions. These findings indicate that the AIR approach effectively supports students' cognitive engagement during the learning process.

In contrast, students in the experimental group II generally demonstrated an understanding of basic concepts and were able to perform mathematical manipulations. However, some students experienced difficulties in providing complete conclusions and solving logarithmic inequalities. This suggests that although the PMR approach helps students relate mathematical concepts to real-life contexts, it may not optimally develop formal reasoning skills. The comparison of students' mathematical reasoning ability between the two groups is illustrated in Figure 1.

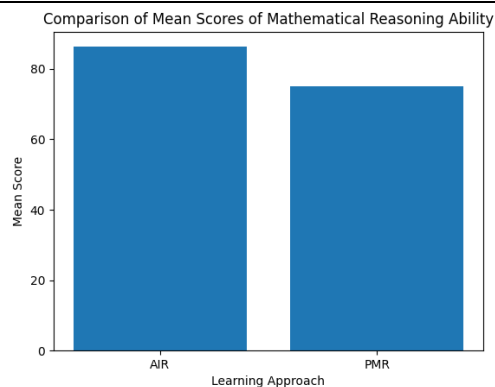


Figure 1. Comparison of Mean Scores of Mathematical Reasoning Ability

The figure shows that the mean score of students' mathematical reasoning ability in the AIR group is higher than that in the PMR group. This difference indicates that learning activities emphasizing auditory engagement, intellectual processing, and repetition in the AIR approach can enhance students' cognitive involvement. Therefore, the AIR approach can be considered an effective alternative in mathematics learning, particularly in improving mathematical reasoning ability.

The results of this study indicate that students taught using the AIR (Auditory, Intellectually, Repetition) approach achieved higher mathematical reasoning scores compared to those taught using the RME approach. This finding is supported by the mean score difference between the two groups, where the AIR group outperformed the PMR group. The statistical analysis using the independent sample t-test also confirmed that the difference was significant at the 5% level. These results suggest that the AIR approach provides a more effective learning environment for developing students' reasoning skills. Therefore, the implementation of AIR contributes positively to improving students' mathematical reasoning ability.

In addition, the descriptive analysis revealed that students in the AIR group demonstrated better performance in several indicators of mathematical reasoning. They were able to present mathematical statements clearly, construct logical arguments, and draw appropriate conclusions. The repetition component in the AIR approach appears to strengthen students' understanding and retention of concepts. Meanwhile, students in the RME group showed good conceptual understanding but faced difficulties in formal reasoning processes. This indicates that different instructional approaches may influence different aspects of students' cognitive development.

These findings are consistent with previous studies that highlight the effectiveness of the AIR approach in enhancing higher-order thinking skills. Prior research has shown that the integration of auditory activities, intellectual engagement, and repetition can significantly improve students' critical thinking and reasoning abilities [22], [23]. Similarly, studies on RME emphasize its strength in connecting mathematics with real-life contexts, which supports conceptual understanding. However, most previous studies examined these approaches separately rather than comparing them directly [24]. This study strengthens existing literature by confirming that AIR may have a stronger impact on reasoning ability compared to RME in certain contexts.

The novelty of this study lies in its direct comparison between AIR and RME approaches within the same research framework. Unlike previous research that focused on a single instructional approach, this study provides empirical evidence on their comparative effectiveness. It also specifically focuses on mathematical reasoning ability as the main outcome variable, which is a crucial competency in mathematics learning [25], [26]. Furthermore, the use of a quasi-experimental design with controlled classroom implementation enhances the validity of the findings [27]. Therefore, this study offers new insights into selecting appropriate instructional strategies for improving students' reasoning skills.

The implications of this study are significant for mathematics educators and curriculum developers. Teachers are encouraged to consider the AIR approach as an alternative strategy to enhance students' reasoning ability in mathematics learning [28]. The structured activities in AIR, including listening, thinking, and repetition, can foster active participation and deeper understanding [29], [30]. Additionally, the findings suggest that combining elements of AIR and RME may further enrich the learning process. Thus, instructional design should be adapted to meet students' cognitive needs and learning characteristics.

Despite its contributions, this study has several limitations that should be considered. The research was conducted in a single school with a limited sample size, which may affect the generalizability of the findings. The study also focused only on one topic, namely logarithmic material, which may not represent other mathematical topics. In addition, external variables such as students' prior knowledge and learning motivation were not fully controlled [31], [32]. Future research is recommended to involve a larger sample and explore

different mathematical topics. Further studies may also investigate the integration of multiple instructional approaches to optimize learning outcomes.

4. CONCLUSION

This study concludes that the expectation outlined in the introduction, namely to identify the comparative effectiveness of the AIR (Auditory, Intellectually, Repetition) and RME (Realistic Mathematics Education) approaches in improving students' mathematical reasoning ability, has been successfully achieved through the findings presented in the results and discussion. The statistical analysis demonstrates that there is a significant difference between the two approaches, with the AIR approach showing higher effectiveness in enhancing students' mathematical reasoning ability. This result is supported by both inferential analysis and descriptive findings, which indicate that students in the AIR group performed better in constructing arguments, performing mathematical manipulations, and drawing logical conclusions. The findings confirm that structured learning activities involving auditory engagement, intellectual processing, and repetition play a crucial role in strengthening students' reasoning skills. Furthermore, this study contributes to the development of instructional strategies by providing empirical evidence that can guide teachers in selecting more effective approaches in mathematics learning. In the future, further research is recommended to explore the integration of AIR with other innovative approaches, as well as its application in different mathematical topics and broader educational contexts to enhance its generalizability and practical impact.

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