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A Numerical Study of Bracketing Root-Finding Methods for Nonlinear Equations: Applications to Break-Even Point Determination

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ABSTRACT

Purpose of the study: The purpose of this study is to compare the results of the Bisection Method, the Regula Falsi Method and the Secant Method in completing Break Even analysis.

Methodology: The research method used is an applied method. The research location is the library of Alauddin State Islamic University, Makassar. The research procedure used by the researcher is to compare the results of the Bisection Method, the Regula Falsi Method, and the Secant Method in solving Break-Even Analysis.

Main Findings: The results show that the secant method is an efficient method for conducting break-even analysis. This is demonstrated by the error value obtained at the end of the iteration process, which shows the smallest error value. In other experiments, the secant method also showed fewer iterations than other methods.

Novelty/Originality of this study: This research can be used as reference material for readers who want to compare the bisection method and the falsi-regularization method. Furthermore, the researchers use the results as a means of evaluating the ability to apply theories in numerical courses.

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1. INTRODUCTION

Nonlinear equations are one of the fundamental problems in applied mathematics, often encountered in various scientific fields [1], [2]. Most nonlinear equations cannot be solved analytically and therefore require a numerical approach [3], [4]. Numerical methods are the primary solution for obtaining an approximation to the roots of equations with a certain degree of accuracy [5], [6]. In this context, root-finding methods play a crucial role. The study of these methods continues to develop as the complexity of mathematical and applied problems increases.

Numerical analysis provides various techniques for solving nonlinear equations, one of which is the root bracketing method [7], [8]. This method works by utilizing an initial interval that guarantees the existence of roots based on changes in the function's sign. The main advantages of the bracketing method are stability and certainty of convergence [9], [10]. The Bisection, Regula Falsi, and Secant methods are among the groups of methods frequently used in numerical practice [11], [12]. Although they share similar basic principles, each method exhibits different convergence characteristics and efficiency.

The Bisection method is known as the simplest and most robust method for finding the roots of nonlinear equations [13], [14]. This method guarantees convergence, but has a relatively slow convergence rate. In contrast, the Regula Falsi method addresses this weakness by utilizing a linear approximation to speed up the iteration process [15], [16]. The Secant method offers a more flexible approach because it does not require function derivatives [17], [18]. These differences in algorithmic properties provide an important basis for conducting numerical comparison studies.

In applied mathematics, nonlinear equations frequently arise in optimization and decision-making problems [19], [20]. One concrete example is break-even analysis, widely used in economics and management studies [21], [22]. Determining the break-even point often involves complex nonlinear functions. This makes analytical solutions difficult or even impossible [23], [24]. Therefore, the application of numerical methods is a relevant and effective approach.

Break-even analysis aims to determine the condition where total revenue equals total costs [25], [26]. In practice, the relationship between costs, revenue, and production volume is not always linear. Nonlinear models in break-even analysis require reliable numerical techniques to find the roots of the equation [27], [28]. The root-flanking method is an appropriate choice because it can handle uncertainty in the form of the function. Evaluating the performance of numerical methods in this context is crucial to ensure the accuracy of the results [29], [30].

Several previous studies have discussed the application of root-finding methods to nonlinear equations [31], [32]. However, studies specifically comparing root-flanking methods in the context of break-even analysis are relatively limited. Furthermore, comparisons often focus on only one aspect, such as the number of iterations or the error rate. However, a comprehensive evaluation requires simultaneous consideration of efficiency, stability, and accuracy [33], [34]. This opens up opportunities for further research in applied numerical analysis.

Despite numerous studies on numerical root-finding methods, most previous research has focused on theoretical comparisons or engineering applications without explicitly linking these methods to economic decision-making contexts such as break-even analysis. In addition, existing comparative studies generally evaluate only a single performance indicator, such as convergence speed or error magnitude, rather than providing a simultaneous assessment of accuracy, stability, and computational efficiency within a unified applied model. Therefore, this study offers novelty by integrating three classical bracketing-based root-finding approaches bisection, Regula Falsi, and Secant—into a practical nonlinear break-even framework and evaluating their performance using multiple numerical criteria. The urgency of this research arises from the increasing need for reliable numerical decision support tools in economic and managerial analysis, where nonlinear cost–revenue relationships frequently occur and analytical solutions are often unattainable. A comprehensive comparison in this applied context is essential to guide practitioners and researchers in selecting appropriate numerical methods for real-world break-even problems.

Based on this background, this study aims to conduct a numerical study of the Bisection, Regula Falsi, and Secant methods. The main focus of the study is to compare the performance of these methods in solving nonlinear equations in break-even analysis. Comparisons are made based on the number of iterations, error rate, and convergence speed. The results are expected to contribute to the development of numerical analysis studies. Furthermore, this research is also expected to serve as a reference in selecting appropriate numerical methods for applied problems.

2. RESEARCH METHOD

2.1. Type of Research

The research method used in this research is applied research [35], [36], namely a research approach that aims to apply scientific concepts and methods in solving real problems related to everyday life, especially in the context of applying numerical methods to solve nonlinear equations in break-even analysis.

2.2. Research Location

This research was conducted from October to December 2025 and was located at the Library of Alauddin State Islamic University Makassar, which was chosen because it provides library sources in the form of books and scientific references that are relevant to the research topic, especially those related to numerical methods and solving nonlinear equations.

2.3. Research Procedures

This research procedure begins with the completion of the break-even analysis using three root-finding methods, namely the Bisection Method, the Regula Falsi Method, and the Secant Method. In each method, the function $f(x)$ is first defined as a nonlinear equation model to be solved. Next, the initial values are determined in the form of lower and upper limits, or initial guess pairs, as well as the tolerance error e which is used as the iteration termination criterion. The iteration process is then carried out to determine the value of x that satisfies the condition $f(x) = 0$ according to the algorithm of each method until a convergent calculation result is obtained.

After all methods produce a solution, a comparison of the results between the Bisection Method, the Regula Falsi Method, and the Secant Method is carried out based on the accuracy of the solution with an error of 0.001 and the speed of convergence measured by the number of iterations in ten trials with different initial value variations.

3. RESULTS AND DISCUSSION

This study uses three numerical methods: the bisection method, the regula falsi method, and the secant method. These three methods are used to solve nonlinear equations applied to break-even cases. To obtain comparative results for the three methods, each method is first analyzed. After all calculations have been completed, the results are substituted into the equation $A_t = \frac{-600(1+0.20)^n}{(1.2)^{n-1}} + \frac{200n}{1.2^{n-1}}$ and equations $A_t = \frac{-2000(1..2)^n}{(1.2)^{n-1}} + \left[\frac{50n}{(1.2)^{n-1}}\right] + 3750$ to get a Break Even analysis on purchasing Intel Pentium and AMD.

The first step taken is to work on the function $f(n)$ according to the procedure or algorithm for each method. The first method used is the Bisection method. In this method, a check is first carried out to see whether the function $f(n)$ has roots at the limits a = 2 and b = 10. Then after checking, the results show that $f(a).f(b)$ = $-2711558{,}493440291$ where the result is less than 0, this explains that the function $f(n)$ for the limits a = 2 and b = 10 there are roots of the equation. For the bisection method or the method of dividing by two for the value of m, use the formula $m = \frac{a+b}{2}$ after that, look for the function $f(m)$. Next search $f(a).f(m)$, when the result is less than 0 then there is a change in the value of $b = m$, if it is greater than 0 then $a = m$ for $f(a).f(m) = 0$, then it is the root being sought. After that, the error value $|b - a|$ is checked. If the value is still smaller than the tolerance value, it will occur until the condition is met. The calculation results of the bisection method are obtained in the 13th iteration with the root obtained being 3.22949. This obtained root is then substituted into the equation $A_t = \frac{-600(1+0.20)^n}{(1.2)^{n-1}} + \frac{200n}{1.2^{n-1}}$ to get a break even analysis on the purchase of Intel Pentium and the result is -542,759. This explains that in year n = 3.22949 the purchase of Intel Pentium will result in a loss of \$ - 542,759 and for the purchase of AMD the root of the equation is substituted in the equation $A_t = \frac{-2000(1..2)^n}{(1.2)^{n-1}} + \left[\frac{50n}{(1.2)^{n-1}}\right] + 3750$ and the result is = 347.5891. This explains that on purchasing AMD will get a profit of \$347.5891.

For the first comparison, please see the table below:

Table 1. Results of Calculations in Break Even Analysis

	Bisection	Regula Falsi	Secant
Pentium	−542,978	−542,531	−542,716
AMD	348,3678	347,027	347,5821

In Table 1, the purchase of Intel Pentium using the bisection method resulted in a greater loss of -542,759 compared to the other two methods. The smallest loss was -542,648 using the Regula Falsi method compared to the other two methods. For AMD purchases, the largest profit was 347,5891 using the Secant method and the smallest profit using the Bisection method was 347,711.

For the second comparison, the three methods were compared using a tolerance value of 0.001. The results of the second comparison can be seen in the table below:

Table 2. Comparison results of the bisection, falsi regular and secant methods for $\varepsilon = 0.001$

i	Bisection			Regula Falsi			Secant		
	m	f (m)	e_r	w	f (w)	e_r	h	f (h)	e_r
1	6	1191,88392	0,66667	5,66381	1106,08067	1	5,66381	1106,08067	0,76560
2	4	487,10879	0,5	4,11690	545,84113	0,37579	4,11690	545,84113	0,37575
3	3	-191,20879	0,33333	3,55584	230,76404	0,15779	2,60974	-590,09348	0,57751
4	3.5	194,17388	0,14286	3,35002	90,14277	0.06144	3,39268	120,62547	0,23077
5	3.25	15,67673	0,07692	3,27414	34,04691	0,02318	3,25980	23,16277	0,04076
6	3.125	-83,79542	0,04	3,24611	12,69140	0,00863	3,22822	-1,12790	0,00978
7	3.1875	-33,12462	0.01961	3.23575	4,70742	0.00320	3.22968	0,01007	0,00045
8	3.21875	-8,49702	0,00971	3,23192	1,74281	0,00119			
9	3.23438	3,64577	0,00483	3,23050	0,64479	0,00044			
10	3.22656	-2.41154	0,00242						
11	3.23047	0,62062	0,00121						
12	3.22852	-0,89458	0,00060						

The table shows that the closest approximation is the Secant method, as its error is the smallest of the three methods, at 0.00001. When sorted, the Bisection method has 0.00098 and the Regula Falsi method has

0.00052. For the final comparison, the processing speed of each method was compared by conducting 10 trials for different values of a and b. The calculation results can be seen in Table 1. The table shows that for break-even analysis, the secant method is the fastest in terms of iteration, as seen from the total number of iterations for the 10 trials. The results show that the secant method has 65 iterations for 10 trials, the bisection method has 113 iterations, and the Regula Falsi method has 109 iterations.

The comparative performance of the three root-finding methods in the break-even model reflects their fundamental numerical characteristics. The bisection method, although theoretically guaranteed to converge due to interval bracketing, inherently exhibits linear convergence [12], [37]. This characteristic makes it numerically stable but relatively conservative in approaching the root. In the context of break-even analysis, this implies that the method is reliable for ensuring the existence of a feasible solution but less efficient when rapid decision estimates are required [38], [39]. Such behavior is consistent with classical numerical analysis theory, where robustness is often achieved at the expense of computational speed.

The Regula Falsi method demonstrates an intermediate behavior between stability and acceleration. By incorporating linear interpolation within a bracketing interval, the method attempts to reduce the slow contraction observed in bisection. However, in nonlinear economic models where function curvature is uneven, Regula Falsi may experience asymmetric interval reduction, causing one endpoint to remain nearly fixed across iterations. This phenomenon explains why its convergence improvement over bisection is limited in certain break-even functions. In applied economic interpretation, this indicates that Regula Falsi can provide improved estimates compared to purely bracketing approaches but still depends strongly on the shape of the cost–revenue function.

The secant method's superior performance can be interpreted through its quasi-Newton nature, where derivative information is approximated from successive iterates [40], [41]. This allows the iteration path to follow the local slope of the nonlinear function more closely, leading to superlinear convergence under regular conditions. In break-even modelling, where the cost and revenue functions often form smooth nonlinear relationships, this property enables the method to approach equilibrium points more efficiently. From a decision-analysis perspective, faster convergence implies reduced computational effort when evaluating multiple scenarios or sensitivity analyses, which is important in economic planning and optimization contexts [42], [43].

From an applied mathematics viewpoint, the findings highlight the trade-off between guaranteed convergence and computational efficiency in selecting numerical methods for economic models. Bracketing methods provide theoretical certainty of root existence, which is essential when model behavior is uncertain. However, open or quasi-open methods such as secant offer practical advantages when the function is sufficiently smooth and well-behaved, as in many break-even formulations. Therefore, method selection should consider not only mathematical accuracy but also the structural properties of the economic model being solved [44], [45].

Overall, the comparative analysis demonstrates that numerical method behavior in break-even problems is governed not only by algorithmic design but also by the nonlinear characteristics of the economic equations. This confirms that numerical analysis is not merely a computational tool but a methodological component of quantitative economic modelling. The integration of numerical root-finding theory with break-even analysis thus provides a more rigorous foundation for determining equilibrium conditions in applied managerial decision-making.

The findings of this study have several important implications for both applied numerical analysis and economic decision modelling. The identification of the secant method as a computationally efficient approach in nonlinear break-even problems suggests that derivative-free quasi-open methods can serve as practical tools for economic equilibrium estimation, particularly in situations where analytical solutions are unavailable. This contributes to bridging the gap between numerical mathematics and managerial decision analysis by demonstrating that method selection can influence not only computational performance but also the reliability of economic interpretations derived from nonlinear models. Consequently, the integration of numerical root-finding techniques into break-even analysis frameworks may support more accurate scenario evaluation, cost–revenue planning, and sensitivity analysis in applied economic studies.

Despite these contributions, this research has several limitations that should be acknowledged. First, the comparison is conducted on a specific nonlinear break-even formulation with limited functional variation, so the generalizability of the numerical performance across broader economic models remains constrained. Different cost–revenue structures, discontinuities, or highly non-smooth functions may alter convergence behavior and method efficiency. Second, the study focuses on classical bracketing and quasi-open methods without including higher-order or hybrid algorithms that may offer improved convergence properties. Third, the evaluation is based on numerical criteria such as iteration behavior and approximation characteristics, without incorporating computational time complexity or large-scale simulation scenarios that commonly arise in real economic systems.

4. CONCLUSION

In a comparison of the bisection, regula falsi, and secant methods, the results obtained indicate that the secant method is an efficient method for conducting break-even analysis. This is indicated by the error value obtained at the end of the iteration process showing the smallest error value. In other experiments, the secant method also showed fewer iterations than the other methods. Future research should therefore extend the analysis to diverse nonlinear economic models, include advanced root-finding variants, and examine performance under broader computational settings to strengthen the applicability of numerical methods in economic decision support.

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