Application of Adams-Bashforth-Moulton Method on Logistic Equation in Predicting Population Growth

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ABSTRACT

Purpose of the study: This study aims to obtain population growth prediction results using the Adams-Bashforth-Moulton method.

Methodology: This type of research is applied research. In this study, the Adams-Bashforth-Moulton method was applied by solving the problem of population growth in South Sulawesi Province.

Main Findings: The logistic equation for population growth with a step size of h = 1 and the capacity of South Sulawesi Province is 20,000,000 people, with a growth rate of 2%. The numerical solution of the logistic equation for population growth at time t = 2020 with an optimal step size of h = 1 is 8,944,168 people.

Novelty/Originality of this study: This research is unique in the form of applying the Adams-Bashforth-Moulton method to solve problems in calculating population growth in South Sulawesi Province.

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1. INTRODUCTION

Mathematics is a basic science that plays an important role in various fields such as engineering, economics, biology, and social sciences. Mathematics provides the tools and frameworks needed to model, analyze, and solve real-world problems [1]-[3]. One significant application of mathematics is through differential equations, which are widely used to describe natural phenomena involving change and growth [4]–[6]. In particular, population dynamics can be effectively modeled using differential equations, especially nonlinear models such as the logistic equation [7]–[9].

The logistic growth model, introduced by Pierre Verhulst, is commonly used to describe population growth that is initially exponential but slows down as the population reaches the carrying capacity of the environment. This model is important for planning in fields such as ecology, demography, and public policy [10]–[12]. However, solving nonlinear differential equations such as the logistic equation analytically is often challenging or even impossible in some cases. Therefore, numerical methods become a practical alternative to obtain approximate solutions [13]–[15].

Numerical methods for solving ordinary differential equations (ODEs) can generally be classified into single-step and multi-step methods. The Adams-Bashforth-Moulton method, which is a multistep predictor-corrector type method, has attracted attention due to its stability and accuracy [16]-[18]. This method combines the explicit Adams-Bashforth method as a predictor with the implicit Adams-Moulton method as a corrector, thus allowing for better error control and convergence properties [19]–[21]. This method is very advantageous when dealing with complex or nonlinear systems where exact solutions are not possible [22]-[24].

Several studies have applied the Adams-Bashforth-Moulton method to various nonlinear problems, such as simple pendulums or projectile motion [25]–[27]. However, there is still a lack of research that applies this method specifically to real-world demographic data, especially in a regional context such as provinces in Indonesia. This gap indicates an opportunity to explore the practical use of the Adams-Bashforth-Moulton method in predicting population growth based on actual regional data.

The urgency of this research lies in the importance of accurate population projections for planning and policy making, especially in rapidly developing areas such as South Sulawesi Province. With estimated carrying capacity and known growth rates, numerical modeling becomes essential to anticipate future demographic trends and guide government resource allocation. The novelty of this study is the application of the Adams-Bashforth-Moulton method to solve the logistic equation for population growth prediction in South Sulawesi Province, using actual regional parameters. Unlike previous studies that focused on abstract or general models, this study bases the numerical method on real demographic data, demonstrating its potential in local forecasting.

Therefore, the main objective of this study is to obtain accurate population growth predictions by applying the Adams-Bashforth-Moulton method to the logistic growth model, using specific data for South Sulawesi Province. This study contributes both methodologically, by validating numerical methods for demographic forecasting, and practically by providing useful predictive insights for regional development planning.

2. RESEARCH METHOD

2.1. Type of Research

Based on the data and results to be achieved, this type of research is application or applied. Applied research is a type of research whose results can be directly applied to solve the problems faced [28]-[30]. In this study, the Adams-Bashforth-Moulton method was applied by solving the problem of population growth in South Sulawesi Province.

2.2. Research Location and Time

In order to obtain data and information in this study, the author chose the Central Statistics Agency of South Sulawesi Province as the place to conduct the research. The time used in this research process was one month.

2.3. Types and Sources of Data

The data used in this study are secondary data. Secondary data is ready-made data [31]-[33]. Data obtained from the Publication of the Central Statistics Agency of South Sulawesi in the form of Population from 2005-2014 in South Sulawesi Province.

2.4. Research Implementation Procedures

The procedures carried out in this study are as follows:

1. Determining the data to be used in the equation
   a. Population data
   b. Growth rate
   c. South Sulawesi's capacity
2. Provides logistic equations.

$P' = \frac{dP}{dt} = m\left(1 - \frac{P}{K}\right)P = f(t, P)$ with initial value $P(t_0) = P_0$ on the interval [a, b] with fixed step size h and $t_{r+1} = t_r + h$

3. Calculate four initial solutions P_0, P_1, P_2, and P_3 using the fourth-order Runge-Kutta method.

$P_{r+1} = P_r + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$ with r = 0, 1, 2.

4. Determine the values of f_r , f_{r-1} , f_{r-2} , f_{r-3} with $r = 3, 4, \dots n$ in the equation:

$P' = \frac{dP}{dt} = m\left(1 - \frac{P}{K}\right)P = f(t, P)$

5. Determining the numerical solution using the Adams-Bashforth method is the predictor equation:

$P_{r+1}^{(0)} = P_r + \frac{h}{24}(55f_r - 59f_{r-1} + 37f_{r-2} - 9f_{r-3})$

6. Calculate $f_{r+1}^{(0)} = f\left(t_{r+1}, P_{r+1}^{(0)}\right)$ and substitute in the Adams-Moulton method:

$P_{r+1} = P_r + \frac{h}{24}\left(f_{r-2} - 5f_{r-1} + 19f_r + 9\, f_{r=1}^{(0)}\right)$

7. The Adams-Moulton correction is iterated on r until it satisfies:

$\frac{\left|P_{r+1}^{(1)} - P_{r+1}^{(0)}\right|}{\left|P_{r+1}^{(1)}\right|} < \varepsilon$

For ε is the desired notification criterion.

8. If the stopping criteria are not met, then an analysis of the selection criteria for the step size h is carried out as follows:
   1) If $\frac{19}{270} \frac{\left|P_{r+1}^{(k)} - P_{r+1}^{(k-1)}\right|}{\left|P_{r+1}^{(k)}\right|} > 10^{-9}$, then h is replaced by $\frac{h}{2}$ and return to step c.
   2) If $\frac{19}{270} \frac{\left|P_{r+1}^{(k)} - P_{r+1}^{(k-1)}\right|}{\left|P_{r+1}^{(k)}\right|} < 10^{-9}$, then h is replaced by 2h and return to step c.
9. If the stopping criteria are met, the iteration continues by returning to step (5) until the specified interval is met.
10. Get a solution to the logistics equation in the South Sulawesi Province area, and find out population growth in the following year.

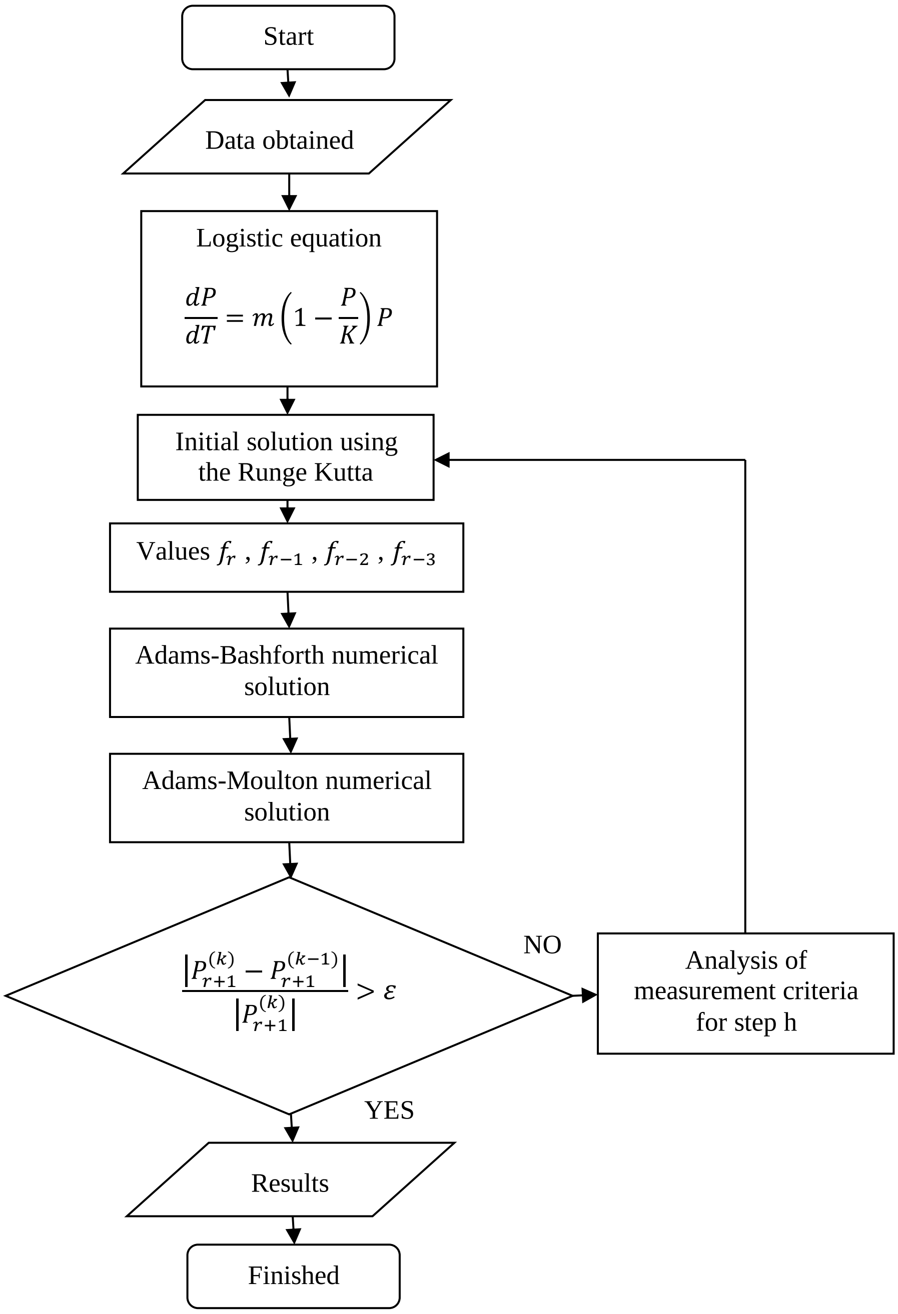

Figure 1. Flowchart

3. RESULTS AND DISCUSSION

3.1. Research result

Based on the research procedure, the steps that need to be taken to obtain the results of predicting population growth in South Sulawesi Province using the Adams-Bashforth-Moulton method. Step 1 The data to be used is population data. Population data can be seen in table 1.

Table 1. Population Data for South Sulawesi Province

No	Year	Census Sesults
1	2005	7,494,701
2	2006	7,629,138
3	2007	7,700,255
4	2008	7,805,024
5	2009	7,908,519

6	2010	8,034,776
7	2011	8,115,638
8	2012	8,250,018
9	2013	8,342,047
10	2014	8,432,163

Source: BPS South Sulawesi Province

Based on Table 1 regarding population data of South Sulawesi Province according to the results of the 2005-2014 Central Statistics Agency census, from this data there was an increase in population as shown in Table 2.

Table 2. Population Growth from 2005-2014

No	Year	Population Increase Every Year
1	2005-2006	134,437
2	2006-2007	71,117
3	2007-2008	104,769
4	2008-2009	103,495
5	2009-2010	126,257
6	2010-2011	80,862
7	2011-2012	134,380
8	2012-2013	92,029
9	2013-2014	90,116

Based on Table 2, it shows that every year the population increases by an average of 104,162 people. Researchers want to predict population growth in the coming year with a logistic equation using the Adams-Bashforth-Moulton method. To determine the growth rate, the following formula is used:

$$m = \frac{1}{t}\ln\left(\frac{P(t)}{P_0}\right)$$

$$m = \frac{1}{t}\ln\left(\frac{7{,}629{,}138}{7{,}494{,}701}\right) = 0.01777 = 0.02$$

Suppose it is assumed that the capacity of South Sulawesi province is $K = 20{,}000{,}000$ with a growth rate of 2%, $P(0) = 7{,}494{,}701$ as the initial value. In the interval [0, 15] With the number of intervals $n = 15$,

$$h = \frac{b-a}{n} = \frac{15-0}{15} = 1$$

with step size $h = 1$

Step 2 Providing the Logistic Equation for Population Growth

The values obtained from step 1 are substituted into the logistic equation

$$\frac{dP}{dt} = m\left(1 - \frac{P}{K}\right)P = 0.02\left(1 - \frac{P}{20{,}000{,}000}\right)P$$

Step 3 Calculate the four initial solutions P_0, P_1, P_2, and P_3 using the fourth-order Runge-Kutta method.

Given $\frac{dP}{dt} = 0.02\left(1 - \frac{P}{20{,}000{,}000}\right)P$ with initial value $P(0) = 7{,}494{,}701$ in the interval [1, 15] with step size $h = 1$.

For $r = 0$ $t_0 = 0$ $P_0 = 7{,}494{,}701$

To calculate the initial solution P_1, first calculate the values of k_1, k_2, k_3, and k_4, as follows.

$$k_1 = hf(t_r, P_r)$$

$$k_1 = hf(t_0, P_0) = hf(0; 7{,}494{,}701)$$

$k_1 = 1\left[0.02\left(1 - \frac{7{,}494{,}701}{20{,}000{,}000}\right)7{,}494{,}701\right] = 93{,}723.476$

$k_2 = hf\left(t_r + \frac{1}{2}h, P_r + \frac{1}{2}k_1\right)$

$k_2 = hf\left(t_0 + \frac{1}{2}h, P_0 + \frac{1}{2}k_1\right)$

$k_2 = hf\left(0 + \frac{1}{2}; 7{,}494{,}701 + \frac{93{,}723.476}{2}\right) = hf(0.5; 7{,}541{,}562.738)$

$k_2 = 1\left[0.02\left(1 - \frac{7{,}541{,}562.738}{20{,}000{,}000}\right)7{,}541{,}562.738\right] = 93{,}956.086$

$k_3 = hf\left(t_r + \frac{1}{2}h, P_r + \frac{1}{2}k_2\right)$

$k_3 = hf\left(t_0 + \frac{1}{2}h, P_0 + \frac{1}{2}k_2\right)$

$k_3 = hf\left(0 + \frac{1}{2}; 7{,}494{,}701 + \frac{93{,}956.086}{2}\right) = hf(0.5; 7{,}541{,}679.043)$

$k_3 = 1\left[0.02\left(1 - \frac{7{,}541{,}679.043}{20{,}000{,}000}\right)7{,}541{,}679.043\right] = 93{,}956.658$

$k_4 = hf(t_r + h, P_r + k_3)$

$k_4 = hf(t_0 + h;\ P_0 + k_3)$

$k_4 = hf(0 + 1;\ 7{,}494{,}701 + 93{,}956.658)$

$k_4 = hf(1;\ 7{,}588{,}657.658)$

$k_4 = 1\left[0.02\left(1 - \frac{7{,}588{,}657.658}{20{,}000{,}000}\right)7{,}588{,}657.658\right] = 94{,}185.428$

After obtaining the values of k_1, k_2, k_3, and k_4 they are then substituted into the fourth order Runge-Kutta equation.

$P_{r+1} = P_r + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

$P_1 = P_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

$P_1 = 7{,}494{,}701 + \frac{1}{6}(93{,}723.476 + 2(93{,}956.086) + 2(93{,}956.658) + 94{,}185.428)$

$P_1 = 7{,}588{,}656.732$

For $r = 1$ obtained $P_1 = 7{,}588{,}656.732$

To calculate the initial solution P_2, first calculate the values of k_1, k_2, k_3, and k_4, as follows.

$t_{r+1} = t_r + h$

$t_1 = t_0 + h$
$t_1 = 0 + 1 = 1$

$k_1 = hf(t_r, P_r)$

$k_1 = hf(t_1, P_1) = hf(1; 7{,}588{,}656.732)$

$k_1 = 1\left[0.02\left(1-\frac{7{,}588{,}656.732}{20{,}000{,}000}\right)7{,}588{,}656.732\right] = 94{,}185.424$

$k_2 = hf\left(t_r + \frac{1}{2}h, P_r + \frac{1}{2}k_1\right)$

$k_2 = hf\left(t_1 + \frac{1}{2}h, P_1 + \frac{1}{2}k_1\right)$

$k_2 = hf\left(1 + \frac{1}{2}; 7{,}588{,}656.732 + \frac{94{,}185.424}{2}\right) = hf(1.5; 7{,}635{,}749.444)$

$k_2 = 1\left[0.02\left(1-\frac{7{,}635{,}749.444}{20{,}000{,}000}\right)7{,}635{,}749.444\right] = 94{,}410.319$

$k_3 = hf\left(t_r + \frac{1}{2}h, P_r + \frac{1}{2}k_2\right)$

$k_3 = hf\left(t_1 + \frac{1}{2}h, P_1 + \frac{1}{2}k_2\right)$

$k_3 = hf\left(1 + \frac{1}{2}; 7{,}588{,}656.732 + \frac{94{,}410.319}{2}\right) = hf(1.5; 7{,}635{,}861.892)$

$k_3 = 1\left[0.02\left(1-\frac{7{,}635{,}861.892}{20{,}000{,}000}\right)7{,}635{,}861.892\right] = 94{,}410.851$

$k_4 = hf(t_r + h, P_r + k_3)$

$k_4 = hf(t_1 + h;\ P_1 + k_3)$

$k_4 = hf(1 + 1;\ 7{,}588{,}656.732 + 94{,}410.851)$

$k_4 = hf(2;\ 7{,}683{,}067.583)$

$k_4 = 1\left[0.02\left(1-\frac{7{,}683{,}067.583}{20{,}000{,}000}\right)7{,}683{,}067.583\right] = 94{,}631.824$

After obtaining the values of k_1, k_2, k_3, and k_4 they are then substituted into the fourth order Runge-Kutta equation.

$P_{r+1} = P_r + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

$P_2 = P_1 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

$P_2 = 7{,}588{,}656.732 + \frac{1}{6}(94{,}185.424 + 2(94{,}410.319) + 2(94{,}410.851) + 94{,}631.824)$

$P_2 = 7{,}683{,}066.664$

For $r = 2$ obtained $P_2 = 7{,}683{,}066.664$

To calculate the initial solution P_3, first calculate the values of k_1, k_2, k_3, and k_4, as follows.

$t_{r+1} = t_r + h$

$t_2 = t_2 + h$

$t_2 = 1 + 1 = 2$

$k_1 = hf(t_r, P_r)$

$k_1 = hf(t_2, P_2) = hf(2; 7{,}683{,}066.664)$

$k_1 = 1\left[0.02\left(1 - \frac{7{,}683{,}066.664}{20{,}000{,}000}\right)7{,}683{,}066.664\right] = 94{,}631.819$

$k_2 = hf\left(t_r + \frac{1}{2}h, P_r + \frac{1}{2}k_1\right)$

$k_2 = hf\left(t_2 + \frac{1}{2}h, P_2 + \frac{1}{2}k_1\right)$

$k_2 = hf\left(2 + \frac{1}{2}; 7{,}683{,}066.664 + \frac{94{,}631.819}{2}\right) = hf(2.5; 7{,}730{,}382.574)$

$k_2 = 1\left[0.02\left(1 - \frac{7{,}730{,}382.574}{20{,}000{,}000}\right)7{,}730{,}382.574\right] = 94{,}843.837$

$k_3 = hf\left(t_r + \frac{1}{2}h, P_r + \frac{1}{2}k_2\right)$

$k_3 = hf\left(t_2 + \frac{1}{2}h, P_2 + \frac{1}{2}k_2\right)$

$k_3 = hf\left(2 + \frac{1}{2}; 7{,}683{,}066.664 + \frac{94{,}843.837}{2}\right) = hf(2.5; 7{,}730{,}491.082)$

$k_3 = 1\left[0.02\left(1 - \frac{7{,}730{,}491.082}{20{,}000{,}000}\right)7{,}730{,}491.082\right] = 94{,}849.329$

$k_4 = hf(t_r + h, P_r + k_3)$

$k_4 = hf(t_2 + h;\ P_2 + k_3)$

$k_4 = hf(2 + 1;\ 7{,}683{,}066.664 + 94{,}849.329)$

$k_4 = hf(3;\ 7{,}777{,}915.993)$

$k_4 = 1\left[0.02\left(1 - \frac{7{,}777{,}915.993}{20{,}000{,}000}\right)7{,}777{,}915.993\right] = 95{,}062.342$

After obtaining the values of k_1, k_2, k_3, and k_4 they are then substituted into the fourth order Runge-Kutta equation.

$P_{r+1} = P_r + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

$P_3 = P_2 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

$P_3 = 7{,}683{,}066.664 + \frac{1}{6}(94{,}631.819 + 2(94{,}843.837) + 2(94{,}849.329) + 95{,}062.342)$

$P_3 = 7{,}777{,}915.079$

Step 4 Calculate the values of $f_r, f_{r-1}, f_{r-2}, f_{r-3}$, with $r = 3, 4, \dots, n$

After getting the initial solution values, then distribute them into the equation $P' = \frac{dP}{dt} = 0.02\left(1 - \frac{P}{20{,}000{,}000}\right)P$, then the following is obtained.

For $r = 3$

$f_r = f_3(t_3 P_3)$

$f_r = f_3(3; 7{,}777{,}915.079)$

$f_r = 0.02\left(1 - \frac{7{,}777{,}915.079}{20{,}000{,}000}\right)7{,}777{,}915.079 = 95{,}062.33861$

$f_{r-1} = f_2(t_2 P_2)$

$f_{r-1} = f_2(2; 7{,}683{,}066.664)$

$f_{r-1} = 0.02\left(1 - \frac{7{,}683{,}066.664}{20{,}000{,}000}\right)7{,}683{,}066.664 = 94{,}631.81992$

$f_{r-2} = f_1(t_1 P_1)$

$f_{r-2} = f_1(1; 7{,}588{,}656.732)$

$f_{r-2} = 0.02\left(1 - \frac{7{,}588{,}656.732}{20{,}000{,}000}\right)7{,}588{,}656.732 = 94{,}185.42365$

$f_{r-3} = f_0(t_0 P_0)$

$f_{r-3} = f_0(0; 7{,}494{,}701)$

$f_{r-3} = 0.02\left(1 - \frac{7{,}494{,}701}{20{,}000{,}000}\right)7{,}494{,}701 = 93{,}732.47692$

Table 3. Initial solution using the Runge-Kutta method on the logistic equation

r	t_r	$h = 1$	
		P_r	$P' = f(t, P)$ $P' = 0.02\left(1 - \frac{P}{20{,}000{,}000}\right)P$
0	1	7,494,701	93,732.47692
1	2	7,588,656.732	94,185.42365
2	3	7,683,066.664	94,631.81992
3	3	7,777,915.079	95,062.33861

Step 5 Determining the numerical solution using the Adams-Bashforth method

After getting the fdr values $f_r(x_r, y_r)$ then substitute them into the Adams-Bashforth equation,

For $r = 3$ $P_3 = 7{,}777{,}915.079$

$t_{r+1} = t_r + h$

$t_4 = t_3 + h$

$t_4 = 3 + 1 = 4$

$P_{r+1}^{(0)} = P_r + \frac{h}{24}(55f_r - 59f_{r-1} + 37f_{r-2} - 9f_{r-3})$

$P_{3+1} = P_3 + \frac{h}{24}(55f_3 - 59f_2 + 37f_1 - 9f_0)$

$P_4^{(0)} = 7{,}777{,}915.079 + \frac{h}{24}\big(55(95{,}062.33861)_r - 59(94{,}631.81992) + 37(94{,}185.42365) - 9(93{,}732.47692)\big) = 7{,}873{,}185.939$

Step 6 Calculate $f_{r+1}^{(0)}$, determine the numerical solution using the Adams-Moulton method and the Adams-Moulton correction until it is satisfied.

After getting the values of $f_{r+1}^{(0)}$ they are then substituted into the Adams-Moulton equation.

$f_4^{(0)}\left(t_4, P_4^{(0)}\right) = f_4(4; 7{,}873{,}185.939)$

$f_4^{(0)}\left(t_4, P_4^{(0)}\right) = 0.02\left(1 - \frac{7{,}873{,}185.939}{20{,}000{,}000}\right)7{,}873{,}185.939 = 95{,}476.66195$

For $r = 3$, $t_4 = 4$ and $P_3 = 7{,}777{,}915.079$

$$P_{3+1}^{(1)} = P_3 + \frac{h}{24}\left(f_1 - 5f_2 + 19f_3 - 9f_4^{(0)}\right)$$

$$P_4^{(1)} = 7{,}777{,}915.079 + \frac{1}{24}\left(94{,}185.42365 - 5(94{,}631.81992) + 19(95{,}062.33861) - 9(95{,}476.66195)\right)$$

$$P_4^{(1)} = 7{,}873{,}185.943$$

The relative error is calculated and compared with the stopping criteria.

$$\varepsilon = 5 \times 10^{-9}$$

$$\frac{\left|P_4^{(1)} - P_4^{(0)}\right|}{\left|P_4^{(1)}\right|} = \frac{|7{,}873{,}185.943 - 7{,}873{,}185.939|}{|7{,}873{,}185.943|} = 4.45674 \times 10^{-10}$$

From these results it can be seen that the relative error is smaller than the stopping criteria.

$$4.45674 \times 10^{-10} < 5 \times 10^{-9}$$

Then the iteration continues until the 15th iteration.

Table 4. Numerical Solution Using the Adams-Bashforth-Moulton Method on the logistic equation for population growth in South Sulawesi Province

Year	t_r	$h = 1$		Relative Error
		$P_r^{(0)}$	P_r (people)	
2005	0		7,494,701	
2006	1		7,588,656.732	
2007	2		7,683,066.664	
2008	3		7,777,915.079	
2009	4	7,873,185.939	7,873,185.943	4.45674×10^{-10}
2010	5	7,968,862.899	7,968,862.903	4.65689×10^{-10}
2011	6	8,064,929.303	8,064,929.303	4.78152×10^{-10}
2012	7	8,161,368.206	8,161,368.21	4.96295×10^{-10}
2013	8	8,258,162.382	8,258,162.386	5.10227×10^{-10}
2014	9	8,355,294.334	8,355,294.338	5.2373×10^{-10}
2015	10	8,452,746.309	8,452,746.313	5.36309×10^{-10}
2016	11	8,550,500.307	8,550,500.311	5.47839×10^{-10}
2017	12	8,648,538.095	8,648,538.1	5.58346×10^{-10}
2018	13	8,746,841.223	8,746,841.228	5.67813×10^{-10}
2019	14	8,845,391.029	8,845,391.035	5.76231×10^{-10}
2020	15	8,944,168.665	8,944,168.67	5.83596×10^{-10}

All numerical solutions in Table 4 have met the stopping criteria $\varepsilon = 5 \times 10^{-9}$. It can be seen that as the years increase, the population in South Sulawesi Province increases.

In this study, the logistic equation is discussed which is a non-linear ordinary differential equation and the population in South Sulawesi Province using the Adams-Bashforth-Moulton method. From the results of the study, in the logistic equation the step size h chosen is 1 with the stopping criterion $\varepsilon = 5 \times 10^{-9}$. In the initial solution using the Runge-Kutta method to find three initial solutions, namely $P_1 = 7{,}588{,}656.732$, $P_2 = 7{,}683{,}066.664$, and $P_3 = 7{,}777{,}915.079$. The three initial solutions are substituted into the logistic equation to obtain the values of f_r, f_{r-1}, f_{r-2}, f_{r-3}. Adams-Bashforth method to predict the value of $P_{r+1}^{(0)}$ with $r = 3, 4, \dots, n$ then the predicted value is corrected using the Adams-Moulton method P_{r+1} with a step size of h = 1. The relative error is smaller than the stopping criterion, so the iteration is continued until the nth iteration. All numerical solutions in Table 3 have met the stopping criteria $\varepsilon = 5 \times 10^{-9}$. It can be seen that as each year increases, the population in South Sulawesi Province increases, as shown in the table below:

Table 5. Population Growth Prediction from 2014-2020

No	Year	Population Growth
1	2014-2015	97,452
2	2015-2016	97,754
3	2016-2017	98,037
4	2017-2018	98,303
5	2018-2019	98,549
6	2019-2020	98,777

Based on Table 5, it shows that every year the population of South Sulawesi Province increases by an average of 98,146.

The results of this study indicate that the Adams-Bashforth-Moulton method is effective in solving nonlinear ordinary differential equations, especially the logistic equation used to predict population growth. By applying this method to population data in South Sulawesi Province, this study is able to produce an estimate of population growth from 2015 to 2020 with a consistent increase every year. The population in South Sulawesi is predicted to increase by around 97,452 to 98,777 people per year within the specified period. These results confirm the suitability of the logistic growth model and the Adams-Bashforth-Moulton method for predicting population trends in a regional context [34]-[36]. The numerical approximation produced by this method is in line with the expected population growth behavior that approaches the environmental carrying capacity [37]-[39].

The implications of this study are very significant, especially in the field of regional planning and policy making. Accurate population projections can help the government and decision makers in designing more effective strategies for infrastructure development, education, health, and resource management. In addition, this study illustrates how numerical methods such as Adams-Bashforth-Moulton can be integrated into applied mathematical modeling to address real-world challenges [40]-[42]. From a methodological perspective, this study supports the use of predictor-corrector approaches to solve nonlinear differential equations that do not have analytical solutions. Thus, this study makes both practical and academic contributions to the development of population modeling techniques.

Despite the positive results, this study has certain limitations. This study assumes a constant growth rate (2%) and carrying capacity (20,000,000), which may not reflect the dynamic socio-economic and environmental factors that influence population changes over time. In addition, the use of a fixed step size ($h = 1$) may affect the accuracy of the results when modeling over a longer time period or under rapidly changing conditions. The model also does not consider migration, variations in birth and death rates, or policy interventions that may significantly alter population trends. These limitations highlight the need for more complex and adaptive models in future research. Furthermore, the implementation is manual or semi-automatic, which may limit scalability.

For future research, it is recommended to explore the application of the Adams-Bashforth-Moulton method to higher-order nonlinear differential equations or systems of differential equations that model more complex population dynamics. Integrating dynamic parameters such as variable growth rates, time-dependent carrying capacities, and migration factors can improve the realism and accuracy of predictions. It is also recommended to develop computational tools or software programs to automate the numerical solution, thereby increasing efficiency and usability for broader applications. Comparative studies involving other multi-step numerical methods such as the Milne-Simpson or Runge-Kutta methods can also provide insight into the relative performance and accuracy of each approach. Through these developments, future research can further bridge the gap between mathematical theory and practical population forecasting.

4. CONCLUSION

Based on the existing objectives and the results obtained from this study, it can be concluded that the Adams-Bashforth-Moulton method on the logistic equation in predicting the population in South Sulawesi Province shows that the population increases every year. Where the results of population growth in South Sulawesi Province in 2015 increased by 97,452 people, in 2016 increased by 97,754 people, in 2017 increased by 98,037 people, in 2018 increased by 98,303 people, in 2019 increased by 98,549 people and in 2020 increased by 98,777 people. This study discusses non-linear first-order ordinary differential equations with their application in everyday life, namely population growth using the Adams-Bashforth-Moulton method. Researchers expect further research on non-linear n-order differential equations in everyday life using the Adams-Bashforth-Moulton method or other multi-step methods and creating programs.

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