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Dynamic Analysis and Stability Evaluation of a Discrete Mathematical Model for Flying Fox String Vibrations

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ABSTRACT

Purpose of the study: This study aims to find the stability of changes in string deflection and the angle of string deflection when an object is launched along a flying fox.

Methodology: This study uses a model built by Kusumastuti, et al. (2017). There are two models analyzed, namely the discrete model of string deflection $y(t)$ and the angle of string deflection $\theta(t)$. The analysis steps include model reduction, model discretization, model linearization, fixed point search, and stability analysis. Stability is analyzed based on its eigenvalues.

Main Findings: Based on the research conducted, the following eigenvalues were obtained: $\lambda_1 = -0{,}005h + (0{,}14565h)i$, $\lambda_2 = -0{,}005h - (0{,}14565h)i$, $\lambda_3 = -0{,}005h + (0{,}1331480769h)i$, dan $\lambda_4 = -0{,}005h - (0{,}1331480769h)i$. The results of the study indicate that the system is in a stable condition (sink) because all eigenvalues obtained are complex negative. Thus, it can be concluded that the string deflection, string deflection velocity, string deflection angle, and string deflection angular velocity are in a stable condition.

Novelty/Originality of this study: This study provides a new contribution to the understanding of discrete system dynamics in flying fox string vibrations, by showing that the stability of the system can be analyzed through the negative complex eigenvalues generated from the model.

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1. INTRODUCTION

Flying fox is one of the popular recreational rides because it challenges the courage and trains the adrenaline of its players [1]-[3]. This ride involves the process of gliding from a height using a rope or steel sling to a lower runway [4]. The popularity of flying fox makes the safety aspect very important in its design [5]-[7]. Based on the National Electronic Injury Surveillance System, there were 16,850 cases of flying fox-related injuries, with 52.3% of them due to falls. The high number of injuries shows the need to improve safety through careful planning [8], [9]. A mathematical approach can be a solution in ensuring the safety of this ride.

Mathematical modeling is an approach to representing physical phenomena into mathematical statements that can be calculated and understood [10]-[12]. In the context of flying fox, mathematical models are used to study the dynamics of vibrations in strings that affect the stability of the ride [13], [14]. This mathematical analysis allows the design of safer and more efficient rides. With a mathematical model, various physical variables that affect vibrations can be identified and evaluated [15]-[17]. This approach is an important basis for overcoming the risk of injury that may occur.

The mathematical model of flying fox string vibrations developed by Kusumastuti [18] uses time-dependent ordinary differential equations. This model formulates the deflection $y(t)$ and the deflection angle $\theta(t)$ to predict the vibration pattern of the string. The numerical simulation results of this model produce graphs of deflection $y_1(t)$, deflection velocity $y_2(t)$, deflection angle $\theta_1(t)$, and deflection angular velocity $\theta_2(t)$. However, the simulation results show several weaknesses, such as too large amplitude and unrealistic vibrations. Therefore, validation of this model is needed to ensure that the results are in accordance with real phenomena. Validation of a mathematical model is an important process to ensure the feasibility of the model in accurately representing physical phenomena [19]-[21]. A valid model must be able to provide consistent, relevant, and representative results for the real system. Validation is carried out to ensure that the model can be implemented safely and efficiently in everyday life. In the context of flying fox, this validation is a key step to improve the safety of the vehicle. By ensuring the accuracy of the model, the risk of accidents can be minimized.

The importance of safety in flying fox systems is not only relevant to technical challenges but also has significant social and economic implications [22]-[24]. Accidents related to flying fox operations can lead to severe injuries, legal liabilities, and reputational damage to operators, all of which underscore the urgency of improving safety measures. Moreover, the growing popularity of flying fox rides worldwide highlights the need for robust designs that can cater to diverse operational conditions and environments [25], [26].

The research conducted by Greefrath [27] highlights the importance of applying mathematical modeling and discrete mathematics in modern mathematics teaching, with a focus on pedagogical aspects and opportunities for developing students' mathematical thinking competencies. Meanwhile, the research by Chand and Koner [28] applies realistic 3D numerical modeling to analyze slope stability and identify failure zones in a mining environment, demonstrating the application of mathematics in a geotechnical engineering context. Both studies emphasize the importance of modeling in their respective domains, but have not specifically examined the dynamics of discrete systems in depth. This study fills this gap by conducting dynamic analysis and stability evaluation of a discrete mathematical model describing the vibrations of the Flying Fox rope. The focus of this study lies in the dynamic system approach, eigenvalue analysis, and system stability, which have not been explicitly the main focus in the two previous studies.

This study has the novelty of developing and analyzing a discrete mathematical model that represents the vibration of a Flying Fox rope, a system that is rarely discussed in the scientific literature, especially from the perspective of system dynamics and stability [29]. This approach not only offers a theoretical understanding of the dynamic behavior of the rope, but also makes practical contributions to the design and safety of Flying Fox systems that are widely used in recreation and agility training. The urgency of this study lies in the importance of ensuring the stability of the rope structure under various operational conditions, which is directly related to the safety of users. By conducting eigenvalue analysis to evaluate the stability of the system, this study provides a strong scientific basis for decision making in the design and evaluation of Flying Fox systems more safely and efficiently.

In addition, this study aims to bridge the gap in the literature by addressing specific shortcomings in previous models, such as unrealistic vibration amplitudes and limited validation methods. By incorporating real-world parameters and testing under various operational conditions, the research seeks to enhance the applicability and reliability of mathematical models for flying fox safety. Based on the explanation above, this study aims to determine the stability of changes in rope deflection and rope deflection angle when an object is launched on a flying fox track.

2. RESEARCH METHOD

2.1. Types of research

The type of research used in this study is qualitative research or literature study. Qualitative research is research that uses data and information that already exists in the literature [24]-[26]. The object of research in this study is the mathematical model of Kusumastuti et al. [18] research on the mathematical model of flying fox string vibrations.

2.2. Pre-Research

The pre-research conducted by the author is to collect a number of references that are in accordance with this research, determine the mathematical model to be used [33], [34]. The author uses a mathematical model constructed by Kusumastuti, et al. [18] to conduct validation analysis. Validation is carried out by conducting a dynamic analysis of the discrete model of flying fox string vibration.

2.3. Research Stages

The stages of this research are carried out through several systematic steps to analyze the mathematical model of flying fox string vibration. First, a model reduction is carried out to simplify the mathematical system so that it is easier to analyze. Furthermore, the reduced model is discretized using the Forward Euler method, which is one of the numerical methods for solving differential equations in discrete form [29]-[31]. After that, the resulting nonlinear model is linearized with Taylor series expansion to facilitate its mathematical analysis [38]. Then, the equilibrium point of the discrete system is determined as a reference in analyzing the stability of the system. Stability analysis is carried out using the Jacobi matrix to calculate the eigenvalues, which are the main indicators of the stability of the equilibrium point [33]-[35]. Finally, simulations are carried out on the discrete dynamic system to observe the behavior of the model as a whole, while interpreting the simulation results to gain further insight into the characteristics of flying fox string vibration.

3. RESULTS AND DISCUSSION

3.1. Dynamic Analysis of Discrete Model of Flying Fox String Vibration

3.1.1. Model Reduction

It is necessary to do a model reduction before conducting stability analysis, so that the model will become a first-order ordinary differential equation system. The mathematical model of flying fox string vibration constructed by Kusumastuti, et al. (2017) is as follows:

$$\frac{d^2y(t)}{dt^2} = -\delta_1 \frac{dy(t)}{dt} - \frac{(\mu_k N + b\eta v - 2EA)y(t)}{m_d L} + \frac{m_b g}{m_d} \quad \ldots (1)$$

$$\frac{d^2\theta(t)}{dt^2} = -\delta_2 \frac{d\theta(t)}{dt} - \frac{6k}{m_d}\left(\frac{\sin\theta\ (t)}{\cos^3\theta\ (t)}\right) + 0.05\ \sin t \quad \ldots (2)$$

Equation (1) is the equation for the deflection of the flying fox string and equation (2) is the equation for the angle of deflection of the flying fox string. Reduction in equation (1) is done by assuming y (t)=P and dP/dt=Q. After the reduction, the system of ordinary differential equations from equation (1) is obtained as follows:

$$\begin{cases} \frac{dP}{dt} = Q \\ \frac{dQ}{dt} = -\delta_1 Q - \frac{(\mu_k N + b\eta v - 2EA)}{m_d L} P + \frac{m_b g}{m_d} \end{cases} \quad \ldots (3)$$

Next, the reduction in equation (2) is carried out by assuming $\theta(t)=R$ and dR/dt=S. After the reduction, the system of ordinary differential equations obtained from equation (2) is as follows:

$$\begin{cases} \frac{dR}{dt} = S \\ \frac{dS}{dt} = -\delta_2 S - \frac{6k}{m_d}\left(\frac{\sin R}{\cos^3 R}\right) + 0{,}05 \sin t \end{cases} \quad \ldots (4)$$

Based on the explanation of the reduction of models (1) and (2) which are in the form of second-order ordinary differential equations, they have been changed into a system of first-order ordinary differential equations, where P is the string deflection equation, Q is the string deflection velocity equation, R is the string deflection angle equation, and S is the flying fox string deflection angular velocity equation.

3.2. Model Discretization

The system of equations (3) is discretized using the standard finite difference method approach for the values $\frac{dP}{dt} = \frac{P_{n+1} - P_n}{h}$ in the equation of string deflection and value $\frac{dQ}{dt} = \frac{Q_{n+1} - Q_n}{h}$ on the equation of the string deflection velocity. Then, the system of equations (4) is discretized using the standard finite difference method approach for the values $\frac{dR}{dt} = \frac{R_{n+1} - R_n}{h}$ on the equation of string angles and values $\frac{dS}{dt} = \frac{S_{n+1} - S_n}{h}$ on the angular velocity equation of the string. Thus, the discrete dynamic system model of the flying fox string deflection is obtained as follows:

$$\begin{cases} P_{n+1} = P_n + hQ_n = f(P^*,Q^*) \\ Q_{n+1} = Q_n - h\delta_1 Q_n - \dfrac{h(\mu_k N + b\eta v - 2EA)}{m_d L} P_n + \dfrac{hm_b g}{m_d} = g(P^*,Q^*) \end{cases} \quad \dots (5)$$

And also obtained the discrete dynamic system model of the flying fox string angle as follows:

$$\begin{cases} R_{n+1} = R_n + hS_n = f(R^*,S^*) \\ S_{n+1} = S_n - h\delta_2 S_n - \dfrac{6kh}{m_d}\left(\dfrac{\sin R_n}{\cos^3 R_n}\right) + 0{,}05 \sin t\,(h) = g(R^*,S^*) \end{cases} \quad \dots (6)$$

The system of equations (5) is a discrete flying fox string vibration deflection model and the system of equations (6) is a discrete flying fox string vibration deflection angle model where h is a positive step size.

3.3. Linearization of the Model

Linearization is carried out if there are non-linear terms in an equation before carrying out equilibrium analysis. The non-linear terms in the two systems of equations are found in equation (6), namely $\frac{sinR_n}{cos^3R_n}$. Linearization is done by expanding the nonlinear terms using a Taylor series around $R_n = 0$ as follows:
The Taylor series formulation is:

$$f\,(x) \approx f\,(c) + \frac{f'(c)}{1!}(x-c) + \frac{f''(c)}{2!}\,(x-c)^2 + \frac{f'''(c)}{3!}(x-c)^3 + \dots \quad \dots (7)$$

The linearization process in this case only requires a Taylor series with a truncation at the second derivative. Thus we obtain:

$$S_{n+1} = S_n - h\delta_s S_n - \frac{6kh}{m_d} R_n \dots (8)$$

Thus, the nonlinear ordinary differential equation has become a linear ordinary differential equation.

3.4. Equilibrium Point of the Model

The discrete dynamic equilibrium point in the system of equations (5) is the point (P^*,Q^*) which is obtained by fulfilling Eq $f(P^*,Q^*) = P^*$ and $g(P^*,Q^*) = Q^*$. In the first case, it is assumed that the deflection and deflection velocity at the initial conditions are zero or $P^* = 0$, because $P^* + hQ^* = P^*$ so $hQ^* = 0$ or $Q^* = 0$. So that the equilibrium point is obtained from the system of equations (5), namely $E_1(P^*,Q^*) = (0,0)$. This equilibrium point is a trivial equilibrium point. So we get the second equilibrium point from the system of equations (5), namely $E_2(P^*, Q^*) = \left(\frac{(m_b g)(L)}{\mu_k N + b\eta v - 2EA}, 0\right)$.

Next, because the system of equations (5) and (6) are uncoupled, each equilibrium point is sought. The discrete dynamic equilibrium point in the system of equations (6) is the point (R^*,S^*) which is obtained by fulfilling Eq $f(R^*,S^*) = R^*$ and $g(R^*,S^*) = S^*$. In the first case, because in the initial conditions the angle and angular velocity do not change, it can be written $R^* = 0$, because $R^* + hS^* = R^*$ so $hS^* = 0$ or $S^* = 0$. So that the equilibrium point is obtained from the system of equations (6), namely $E_1\,(R^*,S^*) = (0,0)$. This equilibrium point is a trivial equilibrium point. So we obtain the second equilibrium point from the system of equations (6), namely $E_2(R^*, S^*) = \left(\frac{0.05\,sin\,t\,(m_d)}{6k}, 0\right)$.

3.5. Stability Analysis at Equilibrium Point

Stage I. Analysis of the stability of the deflection equation. To determine the type of stability of the equilibrium point in the system of equations (5), an eigenvalue analysis of the Jacobian matrix for E_2 (P^*,Q^*) is carried out. Thus, the Jacobi matrix for the system of equations (5) is obtained as follows:

$$\boldsymbol{A_1} = \begin{bmatrix} 1 & h \\ -\dfrac{h(\mu_k N + b\eta v - 2EA)}{m_d L} & 1 - h\delta_1 \end{bmatrix}$$

Next, to find the eigenvalues of the Jacobian matrix at the equilibrium point at E_2, we obtain:

$$\lambda_{1,2} = -\frac{(h\delta_1 - 2) \pm \sqrt{(h\delta_1 - 2)^2 - 4(1)\frac{(1-h\delta_1)m_d L + h^2(\mu_k N + b\eta v - 2EA)}{m_d L}}}{2(1)}$$

$$= -\frac{(h\delta_1 - 2) \pm \sqrt{(h^2\delta_1^2 - 4h\delta_1 + 4) - 4\left(\frac{h^2(\mu_k N + b\eta v - 2EA)}{m_d L}\right)}}{2} \quad \ldots (9)$$

$$= -\frac{1}{2}\frac{\delta_1 h \cdot m_d L - 2m_d L \pm \sqrt{L^2\delta_1^2 h^2 m_d^2 - 4L(\mu_k N + b\eta v - 2EA)h^2 m_d}}{m_d L}$$

Then substitute the parameter values by Kusumastuti, et al. (2017) into $\lambda_{1,2}$, so that the eigenvalues are obtained. $\lambda_1 = -0.005h + (0.14565h)$ i and $\lambda_2 = -0.005h + (0.14565h)i$. Because $\lambda_{1,2} \in \mathbb{C}$, with h always being positive, $\lambda_1 = a + ib$ or $\lambda_2 = a - ib$ and $r = \sqrt{a^2 + b^2}$ so $r = \sqrt{(-0.005)^2 + (0.14565^2} = 0.14574$ if $|r| < 1$ then the equilibrium point $E_2(P^*, Q^*) = \left(\frac{(m_b g)(L)}{\mu_k N + b\eta v - 2EA}, 0\right)$ is stable (sink). So it can be said that the deflection of the string and the speed of the deflection of the string show behavior that is close to the actual state.

Stage II. Analysis of the stability of the equation of the angle of deflection of the string. To determine the type of stability of the equilibrium point in the system of equations (6), an analysis of the eigenvalues of the Jacobian matrix is carried out for $E_2(R^*, S^*)$. Next, for the Jacobian matrix in the system of equations (6) at the equilibrium point $(R^*, S*)$ So the Jacobi matrix for the system of equations (6) is obtained as follows:

$$\boldsymbol{A_2} = \begin{bmatrix} 1 & h \\ -\frac{6kh}{m_d} & 1 - h\delta_2 \end{bmatrix}$$

Next, to find the eigenvalues of the Jacobian matrix at the equilibrium point at E_2, we obtain:

$$\lambda_{3,4} = -\frac{b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= -\frac{(h\delta_2 - 2) \pm \sqrt{(h\delta_2 - 2)^2 - 4(1)\left(\frac{(1 - h\delta_2)m_d + h^2 6k}{m_d}\right)}}{2(1)}$$

$$= -\frac{(h\delta_2 - 2) \pm \sqrt{h^2\delta_2^2 - 4h\delta_2 + 4 - \left(\frac{4(1 - h\delta_2)m_d + 4h^2 6k}{m_d}\right)}}{2}$$

$$= -\frac{1}{2}\frac{h\delta_2 m_d + 2m_d \pm \sqrt{h^2\delta_2^2 m_d^2 - 24h^2 k \cdot m_d}}{m_d} \quad \ldots (10)$$

Then substitute the parameter values by Kusumastuti, et al. (2017) into $\lambda_{3,4}$, so that the eigenvalue is obtained $\lambda_3 = -0.005h + (0.1331480769h)$ i and $\lambda_4 = -0.005h + (0.1331480769h)i$. Because $\lambda_{3,4} \in \mathbb{C}$, with h always being positive, $\lambda_3 = a + ib$ or $\lambda_4 = a - ib$ and $r = \sqrt{a^2 + b^2}$ so $r = \sqrt{(-0.005)^2 + (0.13311480769^2} = 0.1332419$ jika $|r| < 1$ maka titik kesetimbangan $E_2(R^*, S^*) = \left(\frac{0.05 \sin t\,(m_d)}{6k}, 0\right)$ is stable (sink). So it can be said that the angle of deflection of the string and the angular velocity of the deflection of the string show behavior that is close to the actual state.

3.6. Simulation and Interpretation

Simulation I (Flying Fox String Vibration Deflection). Discretely, the simulation can be explained as follows:

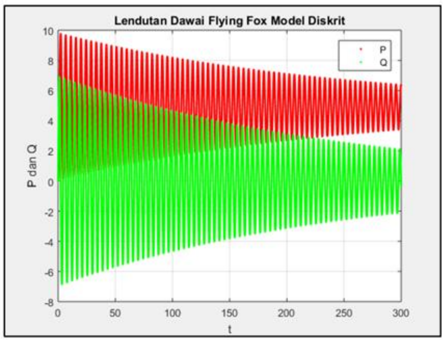

Figure 1. Simulation of the Flying Fox String Deflection Discrete System: initial values for P (0) = 0,Q (0) = $0, \delta_1 = 1/100, m_b = 50, and\ m_d = 50$

Figure 1 shows a numerical simulation of a discrete model of deflection on a flying fox string when $t \in$ [0,300] with initial conditions $P(0) = 0$ and $Q(0) = 0$ as a special requirement of ordinary differential equations. From the graph it can be seen that the simulation shows that the equilibrium point exists and is stable. This can be seen from the direction of the solution towards the equilibrium point. Likewise with the results of the eigenvalues in the form of complex negative numbers. Thus the analyzed model is stable with the parameters used. The graph of the results of solving the continuous model of vibration deflection of flying fox strings numerically using the 4th order Runge-Kutta method will be presented as follows:

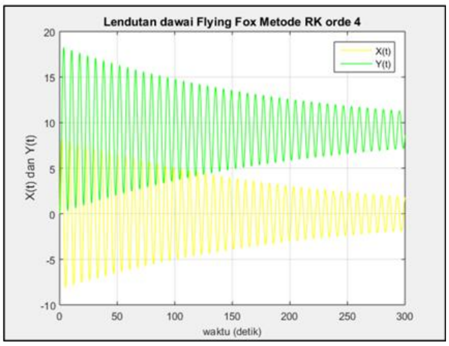

Figure 2. Simulation I of the Flying Fox String Deflection Continuous System: initial values for P (0) = 0,Q (0) = $0, \delta_1 = 1/100, m_b = 50, and\ m_d = 50$

Figure 2 shows a graph of the continuous flying fox string vibration mathematical model. The graph is displayed with the MATLAB program at the first 5 minutes or when $t \in [300]$ in seconds. From the graph it can be seen that the deflection decreases over time, this is in accordance with the phenomenon in the field that the string has a large deflection when the object or load begins to be launched on the flying fox ride. From the discrete and continuous graphs it can be seen that both simulations show that the equilibrium point exists and is stable. This can also be seen from the differences in the graphs which are not too significant.

Simulation II (Flying Fox String Vibration Deflection Angle). Discretely, the simulation can be explained as follows:

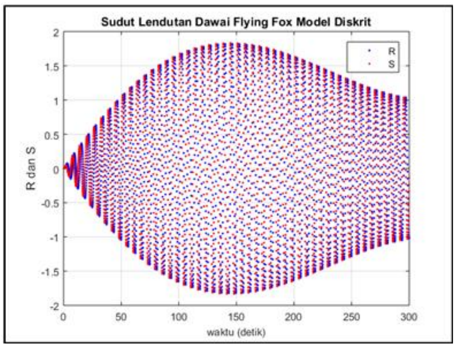

Figure 3. Simulation of the Flying Fox String Deflection Discrete System: initial values for R (0) = 0,S (0) = $0, \delta_2 = 1/100, m_b = 50, and\ m_d = 50$

In Figure 3, it can be seen that the simulation shows that the equilibrium point exists and is stable. This can be seen from the direction of the solution towards the equilibrium point. Likewise, the results of the eigenvalues are in the form of complex negative numbers. Thus, it is concluded that the analyzed model is stable with the parameters used. The graph of the results of solving the continuous model of the vibration angle of the flying fox string numerically using the Runge-Kutta method of order 4 will be presented as follows:

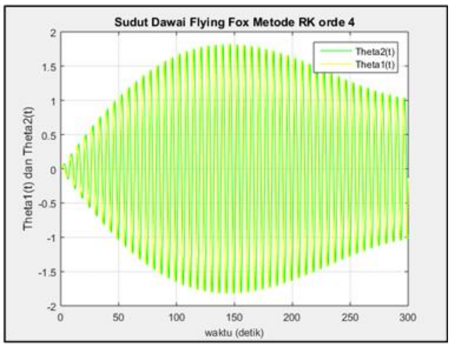

Figure 4. Simulation I of Flying Fox String Deflection Continuous System: initial value for R (0) = 0,S (0) = $0, \delta_2 = 1/100, m_b = 50, and\ m_d = 50$

Figure 4 shows a graph of the mathematical model of the vibration deflection angle of the continuous flying fox string. The graph is displayed with the MATLAB program at the beginning of 5 minutes or when $t \in$ [300] in seconds. From the graph it can be seen that the deflection angle increases over time and then decreases again when the flying fox is about to finish, this is in accordance with the phenomenon in the field that the string has a small deflection angle when the object or load begins to be launched on the flying fox ride. From the discrete and continuous graphs it can be seen that both simulations show that the equilibrium point exists and is stable. This can also be seen from the differences in the graphs which are not too significant.

The results of the discrete model analysis show that the flying fox rope vibration system is in a stable condition, as indicated by the complex eigenvalues with negative real parts. This means that any small disturbance to the system will dampen over time and the system will return to the equilibrium point. This finding confirms that the mathematical model used represents the physical dynamics of the rope well in the operational context of the flying fox.

Furthermore, the model reduction and linearization process is an important step to simplify the second-order differential system into a first-order system, which is easier to analyze numerically [36]-[38]. The use of the Forward Euler method for discretization also shows the efficiency in solving the model numerically and provides results that are in line with the simulation of the continuous model solved by the fourth-order Runge-Kutta method [39]-[41]. The agreement of the results between the discrete and continuous models shows the validity of the discrete approach used.

Numerical simulations performed for both rope deflection and angle of deflection show that the solution direction is toward the equilibrium point. This phenomenon is consistent with the field conditions, where the flying fox rope shows large initial vibrations when the load starts to move, but then dampens gradually. Thus, this model can accurately represent the behavior of the actual physical system.

The important contribution of this study lies in the integration of the discrete dynamic system approach with stability analysis, which is rarely applied in rope-based recreation studies such as flying fox. In addition to enriching the academic literature, this finding can be used as a basis for developing safer flying fox design guidelines, considering that system stability is a key indicator to avoid accidents due to extreme rope fluctuations during operation.

This research has a significant impact on the development of mathematical models for physical systems involving rope dynamics, especially in flying fox rides. With a discrete approach and structured stability analysis, this research provides a scientific basis that can be utilized in the design of safer and more efficient systems [48]-[50]. Practically, this model can be used by engineers or ride managers to evaluate rope behavior in various operational scenarios without having to rely on risky physical trials. However, this research has several limitations. The model used is still based on ideal parameters taken from previous research, so it has not considered external variables such as changes in temperature, wind, or load variations in real time. In addition, the simulation is carried out in two-dimensional space and has not accommodated the complexity of three-dimensional movements that may occur in real rope systems. These limitations provide opportunities for further research to develop more complex and realistic models, including integration with empirical data in the field.

4. CONCLUSION

The conclusion of this study is that the discrete model of flying fox string vibration in the form of a second-order ordinary differential equation is reduced to produce a first-order ordinary differential equation. Discretization is carried out using the Euler Forward method to produce linear and non-linear dynamic systems. Linearization is carried out on non-linear differential equations. In each dynamic system, an equilibrium point is obtained, namely $E_1(P^*,Q^*)$ = (0,0), $E_2(P^*,Q^*) = \left(\frac{(m_b g)(L)}{\mu_k N+b\eta v-2EA}, 0\right)$, 0) and $E_1(R^*,S^*)$ = (0,0), $E_2(R^*,S^*) = \left(\frac{0.05 \sin t\,(m_d)}{6k}, 0\right)$ so that the type of stability can be determined. Equilibrium point $E_2(P^*,Q^*)$ and $E_2(R^*,S^*)$ is stable with complex negative eigenvalues. In the simulation and interpretation section, numerical simulations and phase portraits are carried out using predetermined parameters. The analysis results obtained are in accordance with the discrete and continuous models. This can be seen from the spiral simulation shape with a direction towards the equilibrium point. Further research is suggested to develop the model by considering environmental variables such as wind, temperature, and dynamic variation of load mass. In addition, implementation of the model in three-dimensional simulation and integration with field experimental data can improve the accuracy and relevance of the analysis results.

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