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Fourth Order Runge-Kutta and Gill Methods in Numerical Analysis of Predator-Prey Models

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ABSTRACT

Purpose of the study: This study aims to solve the numerical solution of the Predator-Prey model using the fourth-order Runge-Kutta and Gill methods, and to determine the profile of the Predator-Prey model solved numerically using the fourth-order Runge-Kutta and Gill methods.

Methodology: Schematically, the steps taken in this study are starting from a literature review of the Predator-Prey Model, then solving the Predator-Prey Model using the Fourth-Order Runge-Kutta and Gill Methods, then the program creation step which is continued with program simulation, and finally analysis of the simulation results.

Main Findings: From the results of the analysis of the difference in estimates of the fourth-order Runge-Kutta and Gill for predators and prey, there is no significant difference between the two methods in determining a better method in solving the Predator-Prey model. Because the Predator-Prey model cannot be solved analytically, the difference between the two methods cannot be seen from the analytical solution approach. The simulation results using the fourth-order Runge-Kutta and Gill methods show that the greater the value of b, the prey population increases with a value of $\alpha > \beta$, and the smaller the values of α and β given, the interaction process between the two populations will slow down and the prey population will increase.

Novelty/Originality of this study: can provide information about the profile of the Predator-Prey model which is solved numerically using the fourth-order Runge-Kutta and Gill methods. The combination of these two methods to solve the Predator-Prey model is the novelty of this study.

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1. INTRODUCTION

The role of mathematics has given a very big influence on the progress of knowledge and technology from year to year. Mathematical models are one part of this development, almost all problems in the real world can be formulated into mathematical models [1]-[3]. One of them is about living things on earth. Living things always depend on other living things and consist of various species that form populations [4], [5]. Each individual will always be in contact with other individuals of the same or different types, both individuals in one population or individuals from other populations [6], [7].

The movement of the population, there are several types of relationships that can occur between species. One of these interactions is predation, namely the relationship between prey and predator [8]-[10]. In the discussion of ecological science, especially the interaction of predation between these two populations, it is very important because human survival depends on the balance of the surrounding environment. And this balance can be achieved if the average number of species from the two populations, namely the prey and predator populations (Predator-Prey) that are interacting according to their size or proportion [11]-[13]. The Predator-Prey model was introduced by Alfred J. Lotka and Vito Volterra around 1920, who formulated the mathematical model in a system of differential equations [14]-[16].

The exact solution of the Lotka-Volterra differential equation system explicitly or analytically is not easy to solve, but with numerical methods the equation system can be solved and produce numerical results in the form of approximate solutions [17]-[19]. Several studies have examined the Lotka-Volterra Predator-Prey model, one of which is Vadillo's study [20], which compared the stochastic Lotka–Volterra Predator-Prey model. The study found important differences between three different stochastic models, all of which were developed from a single deterministic Lotka–Volterra model. Ghanbari and Djilali [21] studied the mathematical and numerical analysis of the three-species Predator-Prey model with swarm behavior and fractional time derivatives. This study showed very good agreement between the numerical results and theoretical analysis.

Although various studies have been conducted to explore the Lotka-Volterra model using stochastic approaches and fractional order derivatives, there have been few studies that directly compare the effectiveness of the fourth-order Runge-Kutta and Gill Runge-Kutta methods in solving the model numerically. The fourth-order Runge-Kutta method is widely known for its high accuracy [22]-[24]. However, the Gill Runge-Kutta method, which is also part of the fourth-order Runge-Kutta family, has unique features in constant calculation and computational efficiency. The combination of these two methods to solve the Predator-Prey model has not been widely studied in the literature, so this study offers novelty by filling this gap. In this study, a numerical analysis of the Predator-Prey model will be carried out using the fourth-order Runge-Kutta and Gill methods. The two methods were taken because they have high accuracy.

The urgency of this research lies in the importance of maintaining ecosystem balance through a deeper understanding of predator-prey interactions. Accurate and efficient modeling allows better prediction of population dynamics that can be used for decision making in environmental conservation, natural resource management, and mitigation of environmental change impacts. In this context, more accurate and efficient numerical analysis using the fourth-order Runge-Kutta and Gill methods is becoming increasingly important.

The objectives to be achieved in this study are to solve the numerical solution of the Predator-Prey model using the fourth-order Runge-Kutta and Gill methods, and to determine the profile of the Predator-Prey model solved numerically using the fourth-order Runge-Kutta and Gill methods. This study is expected to provide valuable information regarding the profile of the Predator-Prey model solved numerically using the fourth-order Runge-Kutta and Gill methods. In addition, the results of this study are also expected to be a reference for researchers and practitioners in the fields of ecology and applied mathematics to choose the most appropriate numerical method for their needs.

2. RESEARCH METHOD

Schematically, the steps taken in this research are seen in Figure 1.

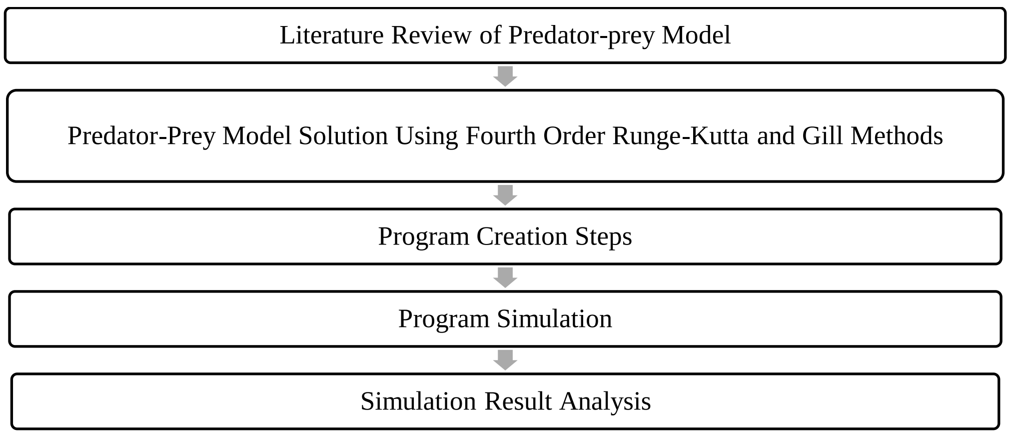


Figure 1. Research Method Diagram

From the scheme in Figure 1, the research steps can be explained as follows.

a. Literature Review of the Predator-Prey Model

Literature review is a study conducted to obtain data and information from books, articles, journals and final assignments related to material on the Predator-Prey model [25]-[27], numerical methods that will be used to solve the Predator-Prey model and study the data that will be taken as well as life tables of prey and predator populations.

Fourth Order Runge-Kutta (RK4) Method

$$x_{i+1} = x_i + \frac{1}{6}h\,(k_1 + 2k_2 + 2k_3 + k_4)h \qquad ...(1)$$

with,

$k_1 = f(t_i, x_i)$

$k_2 = f\left(t_i + \frac{1}{2}h, x_i + \frac{1}{2}hk_1\right)$

$k_3 = f\left(t_i + \frac{1}{2}h, x_i + \frac{1}{2}hk_2\right)$

$k_4 = f\left(t_i + \frac{1}{2}h, x_i + \frac{1}{2}hk_3\right)$

Runge-Kutta Gill (RKG) method

$$x_{i+1} = x_i + \frac{1}{6}(k_1 + k_4) + \frac{1}{3}(sk_2 + uk_3) \qquad ...(2)$$

with,

$k_1 = hf(t_i, x_i)$

$k_2 = hf\left(t_i + \frac{1}{2}h, x_i + \frac{1}{2}k_1\right)$

$k_3 = hf\left(t_i + \frac{1}{2}h, x_i + rk_1 + sk_2\right)$

$k_4 = hf(t_i + h, x_i + tk_2 + uk_3)$

where,

$r = \frac{\sqrt{2}-1}{2}$

$s = \frac{2-\sqrt{2}}{2}$

$t = -\frac{\sqrt{2}}{2}$

$u = 1 + \frac{\sqrt{2}}{2}$

for i = 0, 1, 2, ...,n – 1
n = multiple steps or iterations

Predator Prey Population Model

$$\frac{dx}{dt} = ax - \alpha xy \qquad ...(3)$$

$$\frac{dx}{dt} = bx + \beta xy \qquad ...(4)$$

b. Model solution using the Fourth Order Runge-Kutta and Gill methods

To solve the Predator-Prey model, the fourth-order Runge-Kutta method will be used as in equation (1), and also solved using the Runge-Kutta Gill method, as in equation (2). The Predator-Prey model will be solved by entering the values of the coefficients that have been determined. The coefficients that will be entered include a, α, b, and β which are assumed to lie in the interval [0, 1]. So that the values $0 \le a \le 1$; $0 \le \alpha \le 1$; $0 \le b \le 1$; and $0 \le \beta \le 1$. In addition, the initial values of prey x (0) and predator y (0) that have been determined are also entered. The value of x (0) must be greater than y (0), because in the predation interaction at the beginning of time (t = 0) the number of prey is greater than the number of predators.

c. Program Creation Steps

The software that will be used in creating the program is MATLAB software [28]-[30]. The procedure for creating a simulation program from the Predator-Prey model is as follows:

1) Input parameter values, namely the prey birth rate (a), predator death rate (b), decrease in the prey population (α), increase in the predator population (β), prey population (x), predator population (y) and time (h).
2) Process
   a) create a sub program for the fourth-order Runge-Kutta method.
   b) create a sub program for the Runge-Kutta Gill method.
3) Output.
   The output produced from this simulation is a graph of the number of prey and predator populations against time.

d. Program Simulation

The next step is to simulate several parameters that affect the prey population and predator population by inputting parameter values [31], [32], such as the prey birth rate (a), the predator death rate (b), the decrease in the number of prey populations (α), and the increase in the number of predator populations (β). This simulation is carried out by changing the values of these parameters in the numerical method that will be tried. In this simulation, six cases are varied..

1) α = 0.11, β = 0.0032, a = 0.4, and b = 0.12 ($a > b$)
2) α = 0.11, β = 0.0032, a = 0.4, and b = 0.4 ($a = b$)
3) α = 0.11, β = 0.0032, a = 0.4, and b = 0.74 ($a < b$)
4) α = 0.06, β = 0.0006, a = 0.4, and b = 0.12 ($a > b$)
5) α = 0.06, β = 0.0006, a = 0.4, and b = 0.4 ($a = b$)
6) α = 0.06, β = 0.0006, a = 0.4, and b = 0.74 ($a < b$)

The output produced is a graph of the number of prey and predators against time. In this case, the case study that will be used is the population of green aphids (prey) and red beetles (predators). This data was taken from Angga's research [33] regarding the application of the Lotka-Volterra prey-predator model (case study of coffee and cocoa plantations at PTPN X, and community coconuts in Jember).

e. Simulation Results Analysis

The results obtained from the simulation were then analyzed to determine whether there were changes in the prey and predator populations, namely the green aphid and red beetle populations [34]-[36]. This analysis was carried out by varying the values of the parameters that affect the prey and predator populations. There were four parameters taken, namely the prey birth rate (a), the predator death rate (b), a decrease in the number of prey populations (α), and an increase in the number of predator populations (β). These parameters are likely to affect the number of prey and predators. From the results of the life table calculations, the values of the coefficients a, α, b, and β were obtained, namely 0.40; 0.11; 0.12; and 0.0032. These coefficient values refer to Angga's research [33].

3. RESULTS AND DISCUSSION

Next, we will present the numerical solution of the Predator-Prey model. The Predator-Prey model is a first-order nonlinear differential equation system. Then the first-order nonlinear differential equation system is solved using the fourth-order Runge-Kutta and Gill methods. In the final section, we will discuss the simulation and analysis of the simulation results using the fourth-order Runge-Kutta and Gill methods.

3.1. Research data

The Predator-Prey model is an interaction between two populations, namely the prey and predator populations. The simple Predator-Prey model is defined as the consumption of predators by prey [37], [38]. Based on this model, it can be seen that both species significantly influence each other. Especially if there are abundant prey species, the predator population will also continue to increase. However, conversely, if the growth of the prey species is slow, there will be a decrease in the predator population.

The case study that will be used is the population of green aphids (prey) and red beetles (predators). This data was taken from Angga's research [33] regarding the application of the Lotka-Volterra prey-predator model (case study of coffee and cocoa plantations at PTPN X, and community coconuts in Jember). The green aphids (prey) have a cycle period of between 3-4 months depending on the climate and host plants. Their egg-laying capacity reaches 500 eggs.

3.2. Model Solution Using the Fourth Order Runge-Kutta Method

Equations (3) and (4) will be solved using the fourth-order Runge-Kutta method, in equation (1). The Predator-Prey model to be solved is already a system of equations so there is no need to change it. Equations (3) and (4) are substituted into the fourth-order Runge-Kutta equation as follows.

$$x_{i+1} = x_i + \frac{1}{6}h\,(k_1 + 2k_2 + 2k_3 + k_4) \qquad ...(5)$$

$$y_{i+1} = y_i + \frac{1}{6}h\,(l_1 + 2l_2 + 2l_3 + l_4) \qquad ...(6)$$

with,

$$k_1 = f(t_1, x_1, y_1)$$

$$k_1 = ax - \alpha xy$$

$$l_1 = g(t_1, x_1, y_1)$$

$$l_1 = -by + \beta xy$$

$$k_2 = f\left(t_2 + \frac{h}{2}, x_2 + k_1\frac{h}{2}, y_2 + l_1\frac{h}{2}\right)$$

$$k_2 = a\left(x_2 + k_1\frac{h}{2}\right) - \alpha\left(x_2 + k_1\frac{h}{2}\right)\left(y_2 + l_1\frac{h}{2}\right)$$

$$l_2 = g\left(t_2 + \frac{h}{2}, x_2 + k_1\frac{h}{2}, y_2 + l_1\frac{h}{2}\right)$$

$$l_2 = -b\left(y_2 + l_1\frac{h}{2}\right) - \beta\left(x_2 + k_1\frac{h}{2}\right)\left(y_2 + l_1\frac{h}{2}\right)$$

$$k_3 = f\left(t_3 + \frac{h}{2}, x_3 + k_2\frac{h}{2}, y_3 + l_2\frac{h}{2}\right)$$

$$k_3 = a\left(x_3 + k_2\frac{h}{2}\right) - \alpha\left(x_3 + k_2\frac{h}{2}\right)\left(y_3 + l_2\frac{h}{2}\right)$$

$$l_3 = g\left(t_3 + \frac{h}{2}, x_3 + k_2\frac{h}{2}, y_3 + l_2\frac{h}{2}\right)$$

$$l_3 = -b\left(y_3 + l_2\frac{h}{2}\right) + \beta\left(x_3 + k_2\frac{h}{2}\right)\left(y_3 + l_2\frac{h}{2}\right)$$

$$k_4 = f(t_4 + h, x_4 + k_3h, y_4 + l_3h)$$

$k_4 = a(x_4 + k_3 h) - \alpha(x_4 + k_3 h)(y_4 + l_3 h)$

$l_4 = g(t_4 + h, x_4 + k_3 h, y_4 + l_3 h)$

$l_4 = -b(y_4 + l_3 h) + \beta(x_4 + k_3 h)(y_4 + l_3 h)$

By substituting equations (3) and (4) into equation (1), the following equation is obtained,

$$x_{i+1} = x_i + \frac{1}{6}h\left((ax - \alpha xy) + 2\left(a\left(x_2 + k_1\frac{h}{2}\right) - \alpha\left(x_2 + k_1\frac{h}{2}\right)\left(y_2 + l_1\frac{h}{2}\right)\right) + 2\left(a\left(x_3 + k_2\frac{h}{2}\right) - \alpha\left(x_3 + k_2\frac{h}{2}\right)\left(y_3 + l_2\frac{h}{2}\right)\right) + \left(a(x_4 + k_3 h) - \alpha(x_4 + k_3 h)(y_4 + l_3 h)\right)\right)$$

$$y_{i+1} = y_i + \frac{1}{6}h\left((-by + \beta xy) + 2\left(-b\left(y_2 + l_1\frac{h}{2}\right) - \beta\left(x_2 + k_1\frac{h}{2}\right)\left(y_2 + l_1\frac{h}{2}\right)\right) + 2\left(-b\left(y_3 + l_2\frac{h}{2}\right) + \beta\left(x_3 + k_2\frac{h}{2}\right)\left(y_3 + l_2\frac{h}{2}\right)\right) + \left(-b(y_4 + l_3 h) + \beta(x_4 + k_3 h)(y_4 + l_3 h)\right)\right)$$

3.3. Model Solution Using Runge-Kutta Gill Method

Equations (3) and (4) will be solved using the fourth order Runge-Kutta method, in equation (2). The Predator-Prey model to be solved is already a system of equations so there is no need to change it. Equations (3) and (4) are substituted into the Runge-Kutta Gill equation as follows.

$$x_{i+1} = x_i + \frac{1}{6}(k_1 + k_4) + \frac{1}{3}\left(\frac{2-\sqrt{2}}{2}k_2 + (1 + \frac{\sqrt{2}}{2})k_3\right) \qquad ...(7)$$

$$y_{i+1} = y_i + \frac{1}{6}(l_1 + l_4) + \frac{1}{3}\left(\frac{2-\sqrt{2}}{2}l_2 + (1 + \frac{\sqrt{2}}{2})l_3\right) \qquad ...(8)$$

with,

$k_1 = hf(t_1, x_1, y_1)$

$k_1 = h(ax - \alpha xy)$

$l_1 = hg(t_1, x_1, y_1)$

$l_1 = h(-by + \beta xy)$

$k_2 = hf\left(t_2 + \frac{h}{2}, x_2 + \frac{1}{2}k_1, y_2 + \frac{1}{2}l_1\right)$

$k_2 = h\left(a\left(x_2 + \frac{1}{2}k_1\right) - \alpha\left(x_2 + \frac{1}{2}k_1\right)\left(y_2 + \frac{1}{2}l_1\right)\right)$

$l_2 = hg\left(t_2 + \frac{h}{2}, x_2 + \frac{1}{2}k_1, y_2 + \frac{1}{2}l_1\right)$

$l_2 = h\left(-b\left(y_2 + \frac{1}{2}l_1\right) - \beta\left(x_2 + \frac{1}{2}k_1\right)\left(y_2 + \frac{1}{2}l_1\right)\right)$

$k_3 = hf\left(t_3 + \frac{h}{2}, x_3 + \frac{\sqrt{2}-1}{2}k_1 + \frac{2-\sqrt{2}}{2}k_2, y_3 + \frac{\sqrt{2}-1}{2}l_1 + \frac{2-\sqrt{2}}{2}l_2\right)$

$k_3 = h\left(a\left(x_3 + \frac{\sqrt{2}-1}{2}k_1 + \frac{2-\sqrt{2}}{2}k_2\right) - \alpha\left(x_3 + \frac{\sqrt{2}-1}{2}k_1 + \frac{2-\sqrt{2}}{2}k_2\right)\left(y_3 + \frac{\sqrt{2}-1}{2}l_1 + \frac{2-\sqrt{2}}{2}l_2\right)\right)$

$l_3 = hg\left(t_3 + \frac{h}{2}, x_3 + \frac{\sqrt{2}-1}{2}k_1 + \frac{2-\sqrt{2}}{2}k_2, y_3 + \frac{\sqrt{2}-1}{2}l_1 + \frac{2-\sqrt{2}}{2}l_2\right)$

$l_3 = h\left(-b\left(y_3 + \frac{\sqrt{2}-1}{2}l_1 + \frac{2-\sqrt{2}}{2}l_2\right) + \beta\left(x_3 + \frac{\sqrt{2}-1}{2}k_1 + \frac{2-\sqrt{2}}{2}k_2\right)\left(y_3 + \frac{\sqrt{2}-1}{2}l_1 + \frac{2-\sqrt{2}}{2}l_2\right)\right)$

$$k_4 = hf\left(t_4 + \frac{h}{2}, x_4 + \left(\frac{-\sqrt{2}}{2}\right)k_2 + \left(1+\frac{\sqrt{2}}{2}\right)k_3, y_4 + \left(\frac{-\sqrt{2}}{2}\right)l_2 + \left(1+\frac{\sqrt{2}}{2}\right)l_3\right)$$

$$k_4 = h\left(a\left(x_4 + \left(\frac{-\sqrt{2}}{2}\right)k_2 + \left(1+\frac{\sqrt{2}}{2}\right)k_3\right) - \alpha\left(x_4 + \left(\frac{-\sqrt{2}}{2}\right)k_2 + \left(1+\frac{\sqrt{2}}{2}\right)k_3\right)\left(y_4 + \left(\frac{-\sqrt{2}}{2}\right)l_2 + \left(1+\frac{\sqrt{2}}{2}\right)l_3\right)\right)$$

$$l_4 = hg\left(t_4 + \frac{h}{2}, x_4 + \left(\frac{-\sqrt{2}}{2}\right)k_2 + \left(1+\frac{\sqrt{2}}{2}\right)k_3, y_4 + \left(\frac{-\sqrt{2}}{2}\right)l_2 + \left(1+\frac{\sqrt{2}}{2}\right)l_3\right)$$

$$l_4 = h\left(-b\left(y_4 + \left(\frac{-\sqrt{2}}{2}\right)l_2 + \left(1+\frac{\sqrt{2}}{2}\right)l_3\right) + \beta\left(x_4 + \left(\frac{-\sqrt{2}}{2}\right)k_2 + \left(1+\frac{\sqrt{2}}{2}\right)k_3\right)\left(y_4 + \left(\frac{-\sqrt{2}}{2}\right)l_2 + \left(1+\frac{\sqrt{2}}{2}\right)l_3\right)\right)$$

By substituting equations (3) and (4) into equation (2), the following equation is obtained,

$$\begin{aligned} x_{i+1} = {} & x_i + \frac{1}{6}\Bigg(\big(h(ax - \alpha xy)\big) + \Bigg(h\bigg(a\left(x_4 + \left(\frac{-\sqrt{2}}{2}\right)k_2 + \left(1+\frac{\sqrt{2}}{2}\right)k_3\right) - \alpha\bigg(x_4 + \left(\frac{-\sqrt{2}}{2}\right)k_2 + \bigg(1 + \\ & \frac{\sqrt{2}}{2}\bigg)k_3\bigg)\left(y_4 + \left(\frac{-\sqrt{2}}{2}\right)l_2 + \left(1+\frac{\sqrt{2}}{2}\right)l_3\right)\bigg)\Bigg)\Bigg) + \frac{1}{3}\Bigg(\frac{2-\sqrt{2}}{2}\bigg(h\bigg(a\left(x_2 + \frac{1}{2}k_1\right) - \alpha\left(x_2 + \frac{1}{2}k_1\right)\bigg(y_2 + \\ & \frac{1}{2}l_1\bigg)\bigg)\bigg) + \left(1 + \frac{\sqrt{2}}{2}\right)\bigg(h\bigg(a\left(x_3 + \frac{\sqrt{2}-1}{2}k_1 + \frac{2-\sqrt{2}}{2}k_2\right) - \alpha\left(x_3 + \frac{\sqrt{2}-1}{2}k_1 + \frac{2-\sqrt{2}}{2}k_2\right)\bigg(y_3 + \frac{\sqrt{2}-1}{2}l_1 + \\ & \frac{2-\sqrt{2}}{2}l_2\bigg)\bigg)\bigg)\Bigg) \end{aligned}$$

$$\begin{aligned} y_{i+1} = {} & y_i + \frac{1}{6}\Bigg(\big(h(-by + \beta xy)\big) + \Bigg(h\bigg(-b\left(y_4 + \left(\frac{-\sqrt{2}}{2}\right)l_2 + \left(1+\frac{\sqrt{2}}{2}\right)l_3\right) + \beta\bigg(x_4 + \left(\frac{-\sqrt{2}}{2}\right)k_2 + \\ & \left(1 + \frac{\sqrt{2}}{2}\right)k_3\bigg)\left(y_4 + \left(\frac{-\sqrt{2}}{2}\right)l_2 + \left(1+\frac{\sqrt{2}}{2}\right)l_3\right)\bigg)\Bigg)\Bigg) + \frac{1}{3}\Bigg(\frac{2-\sqrt{2}}{2}\bigg(h\bigg(-b\left(y_2 + \frac{1}{2}l_1\right) - \beta\left(x_2 + \frac{1}{2}k_1\right)\bigg(y_2 + \\ & \frac{1}{2}l_1\bigg)\bigg)\bigg) + \left(1 + \frac{\sqrt{2}}{2}\right)\bigg(h\bigg(-b\left(y_3 + \frac{\sqrt{2}-1}{2}l_1 + \frac{2-\sqrt{2}}{2}l_2\right) + \beta\left(x_3 + \frac{\sqrt{2}-1}{2}k_1 + \frac{2-\sqrt{2}}{2}k_2\right)\bigg(y_3 + \frac{\sqrt{2}-1}{2}l_1 + \\ & \frac{2-\sqrt{2}}{2}l_2\bigg)\bigg)\bigg)\Bigg) \end{aligned}$$

3.4. Program View

The software used to solve the Predator-Prey model is Matlab 7.8.0. Figure 2 shows the GUI display for the Predator-Prey model solution program.

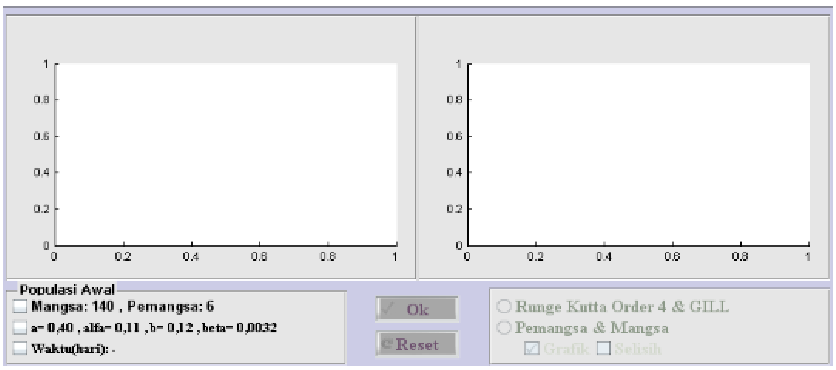


Figure 2. GUI view of the Predator-Prey program

The inputs from the program above include the prey birth rate (a), predator death rate (b), decrease in the prey population (α), increase in the predator population (β), prey population (x), predator population (y). The values of the coefficients a, α, b, and β used are 0.40; 0.11; 0.12; and 0.0032. The GUI display above, in the Checkbox component for prey, predator, the relevant coefficients, and time (days) can be written the coefficient values according to the varied values. Then processed by selecting the fourth-order Runge-Kutta and Gill radiobutton components, then the GUI will display a graph of the results of solving the Predator-Prey model with the fourth-order Runge-Kutta and Gill methods.

The estimated graph in the GUI display above is used to display the graph of the results of solving the Predator-Prey with the fourth-order Runge-Kutta and Gill methods. Other information from the program above includes:

a. OK button, used to find the results of the Predator-Prey solution using the fourth-order Runge-Kutta and Gill methods.

b. Reset button, used to restart the solution process from the beginning.

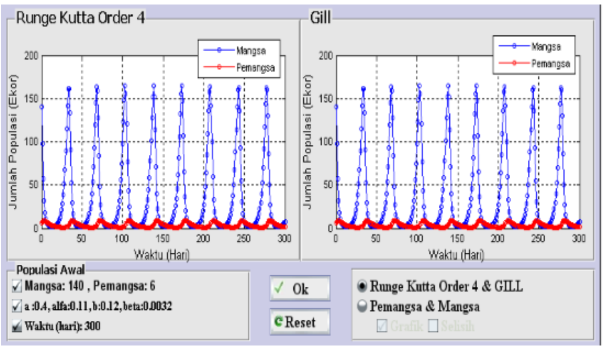
3.5. Program Simulation

In this study, the Predator-Prey model was simulated using the fourth-order Runge-Kutta and Gill methods. The simulation of the Predator-Prey interaction completed using the fourth-order Runge-Kutta and Gill methods was carried out by varying the parameters that affect the Predator-Prey interaction. Some of the parameters that will be varied are the prey birth rate (a), the predator death rate (b), the decrease in the number of prey populations (α) and the increase in the number of predator populations (β). To analyze the effect of these parameters on the Predator-Prey interaction, parameter variations will be carried out by changing the values of four parameters, namely the prey birth rate (a), the predator death rate (b), the decrease in the number of prey populations (α) and the increase in the number of predator populations (β) obtained from the calculation of the life table.

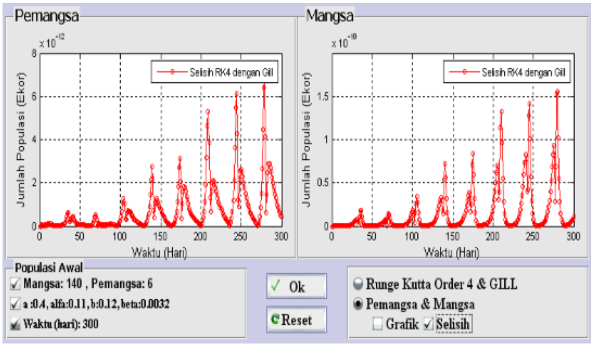
In this study, several cases were given to vary the values of these parameters. There are six cases that will be simulated from the Predator-Prey model using the fourth-order Runge-Kutta and Gill methods.

a. First Case

The value of $\alpha \approx 30\beta$, namely $\alpha = 0.11$ and $\beta = 0.0032$. For the first case simulation, the value of the prey birth rate is given more than the predator death rate ($a > b$), namely 0.4 and 0.12. By using the fourth-order Runge-Kutta and Gill methods, the Predator-Prey model simulation can be seen in the following graph. This graph will later be analyzed to determine the changes that occur in the number of predator and prey populations.



(a) Predator-Prey graph for the first case with $a > b$ ($\alpha \approx 30\beta$)



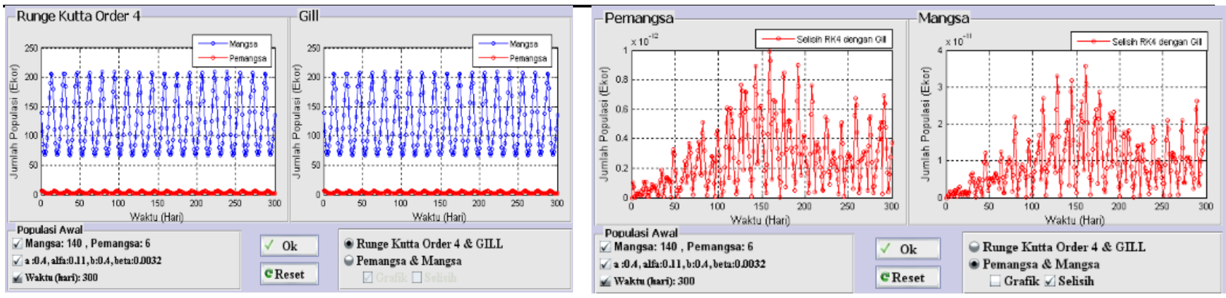
(b) Graph of the difference in estimates with RK4 and Gill for predators and prey

Figure 3. Predator-Prey graph for the first case for $a > b$ values and the difference between the two methods.

Figure 3 (a) shows the Predator-Prey graph for the first case for the values $\alpha = 30\beta$ and $a > b$ with $\alpha = 0.11$; $\beta = 0.0032$; $a = 0.4$; and $b = 0.12$. Meanwhile, Figure 3 (b) shows the graph of the difference in estimates using the fourth-order Runge-Kutta and Gill methods for predators and prey.

b. Second Case

Given the value of the prey birth rate equal to the predator death rate ($a = b$), namely the values $\alpha = 0.11$; $\beta = 0.0032$; $a = 0.4$; and $b = 0.12$ are shown in Figure 4 (a) and the graph of the difference in estimates between the two methods seen from the results of the number of prey and predator populations is shown in Figure 4 (b).



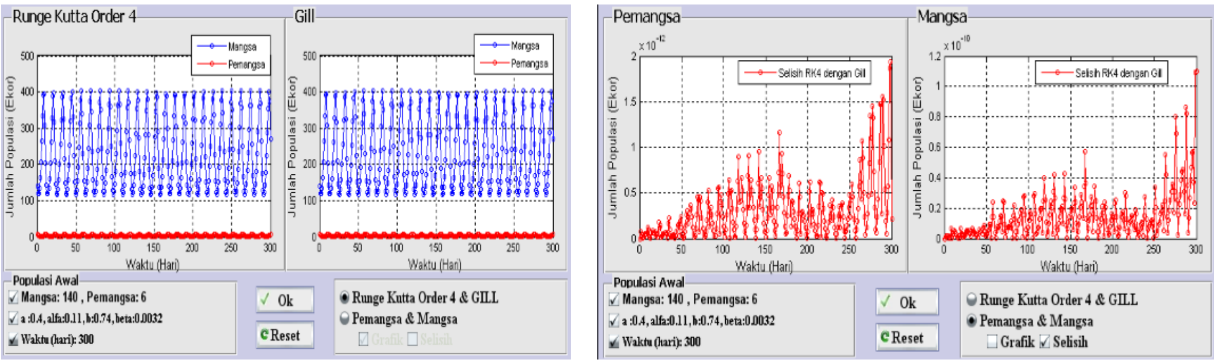
(a) Predator-Prey graph for the second case with a = b (α ≈ 30β)

(b) Estimation difference graph with RK4 and Gill for predators and prey

Figure 4. Predator-Prey graph for the second case for the value a = b and the difference between the two methods

c. Third Case

Given the value of the prey birth rate is less than the predator death rate (a < b), namely the values α = 0.11; β = 0.0032; a = 0.4; and b = 0.74 are shown in Figure 5 (a) and the graph of the difference in estimates of the two methods for predators and prey is shown in Figure 5 (b).



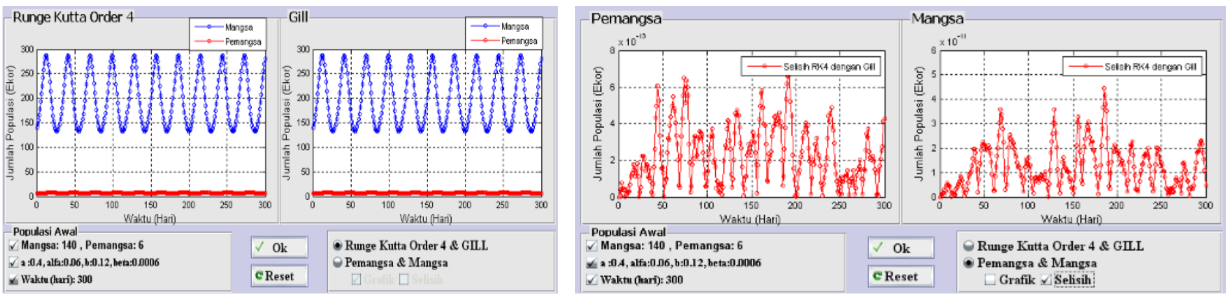
(a) Predator-Prey graph for the first case with a < b (α ≈ 30β)

(b) Graph of the difference between RK4 and Gill estimates for predators and prey

Figure 5. Predator-Prey graph for the third case for a < b values and the difference between the two methods

d. Fourth Case

The value of α = 100β is α = 0.06 and β = 0.0006 in the same way given the value of the prey birth rate is more than the predator death rate (a > b). with the value of a = 0.4; and b = 0.12 is shown in Figure 6 (a) and the graph of the difference in estimates of the two methods for predators and prey is shown in Figure 6 (b).



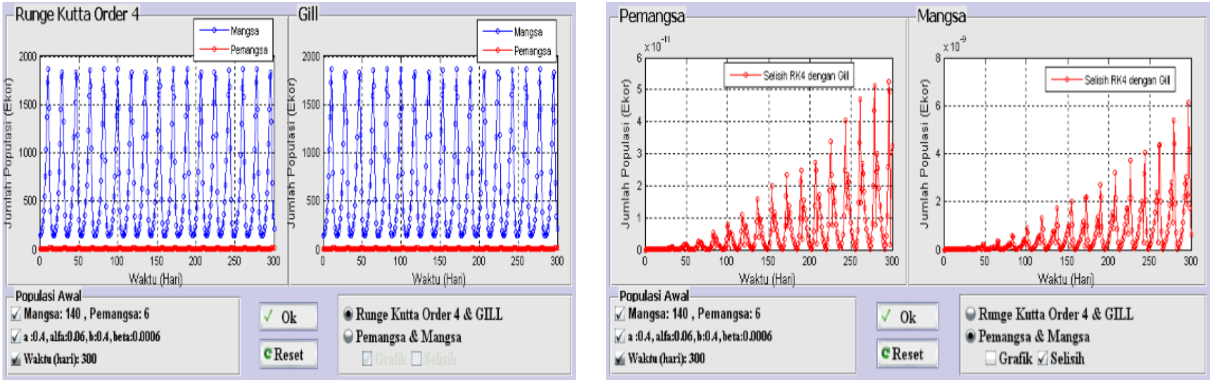
(a) Predator-Prey graph for the second case with a > b (α = 100β)

(b) Graph of the difference between RK4 and Gill estimates for predators and prey

Figure 6. Predator-Prey graph for the fourth case for a > b values and the difference between the two methods.

e. Fifth Case

Given the value of the prey birth rate is the same as the predator death rate (a = b). The fifth case with a = b, given the variation of the values α = 0.06; β = 0.0006; a = 0.4; and b = 0.4 is shown in Figure 7 (a) and the graph of the difference in estimates of the two methods for predators and prey is shown in Figure 7 (b).



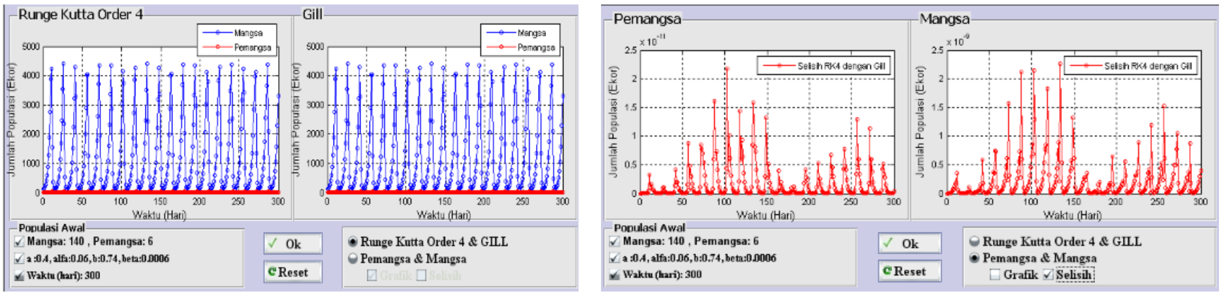
(a) Predator-Prey graph for the second case with $a = b$ ($\alpha = 100\beta$)

(b) Graph of the difference between RK4 and Gill estimates for predators and prey

Figure 7. Predator-Prey graph for the fifth case for the value $a = b$ and the difference between the two methods.

f. Sixth Case

Given the value of the prey birth rate is less than the predator death rate ($a < b$) with a value of α = 0.06; β = 0.0006; a = 0.4; and b = 0.74 is shown in Figure 8 (a) and a graph of the difference in estimates between the two methods for predator and prey is shown in Figure 8 (b).



(a) Predator-Prey graph for the second case with $a < b$ ($\alpha = 100\beta$)

(b) Graph of the difference between RK4 and Gill estimates for predators and prey

Figure 8. Predator-Prey graph for the sixth case for $a < b$ values and the difference between the two methods.

3.6. Simulation Result Analysis

Simulation analysis of the Predator-Prey model with the fourth-order Runge-Kutta and Gill methods that have been carried out. In Figure 3 (a) it can be explained that by giving the values α = 0.11; β = 0.0032; a = 0.4; and b = 0.12 using the fourth-order Runge-Kutta and Gill methods, the resulting graph looks decreasing, as does the predator graph. Then the prey graph is seen increasing, the predator graph also increases. From these results it can be seen that the predator population will increase when the prey population is large. This causes predation pressure to increase, thereby reducing the prey population. A decrease in the prey population will cause the predator population to decrease due to lack of food and predation pressure decreases so that the prey population increases. This is because the prey birth rate is greater than the predator death rate ($a > b$).

In Figure 4 (a) it can be explained that by giving the values α = 0.11; β = 0.0032; a = 0.4; and b = 0.4 using the fourth-order Runge-Kutta and Gill methods, the resulting graph looks like it increases and decreases like the previous image but the predator population is very low. This is because the prey birth rate is the same as the predator death rate ($a = b$). From these results it can be seen that the larger the value, the prey population increases with the values α and β remaining the same.

Figure 5 (a) can be explained that by giving the values α = 0.11; β = 0.0032; a = 0.4; and b = 0.74 using the fourth-order Runge-Kutta and Gill methods, the resulting graph also looks like it increases and decreases. This condition tends to be stable. From the simulation results it can be seen that when the predator death rate is large ($a < b$), the prey population continues to increase. This will cause predation pressure to decrease.

Figure 6 (a) can be explained that by giving the value ($\alpha = 100\beta$), namely α = 0.06; β = 0.0006; a = 0.4; and b = 0.12 using the fourth-order Runge-Kutta and Gill methods, the resulting graphs appear to continue to increase and decrease for the prey and predator populations. From the graph, the interaction process between the two populations slows down and the population is also low. This is because the values of α and β are smaller than the previous cases. From the variation in the values of the coefficients, it can be seen that the prey population increases because the birth rate is large and predation pressure decreases.

Figure 7 (a) can be explained that by giving the values α = 0.06; β = 0.0006; a = 0.4; and b = 0.4 using the fourth-order Runge-Kutta and Gill methods, the resulting graph looks like it is increasing and decreasing like the previous image but the prey population is very high. This is because the prey birth rate is the same as the predator death rate ($a = b$) and the increase in the number of prey is greater than the predator ($\alpha > \beta$). The prey population reproduces very quickly than the predator so that even though the prey birth rate and the predator death rate are the same, the prey population continues to increase.

In Figure 8 (a) it can be explained that by giving the values α = 0.06; β = 0.0006; a = 0.4; and b = 0.74 using the fourth-order Runge-Kutta and Gill methods, the resulting graph looks increasingly increasing for the prey population. This condition tends to be stable. From the simulation results it can be seen that when the predator death rate is large ($a < b$), the prey population continues to increase. This will cause predation pressure to decrease. This is because the death of the predator population is greater than the birth of the prey population ($b > \alpha$). Although the birth of the prey population is smaller, predators have decreased, so the number of prey populations has increased.

The graphic image showing the difference in estimates with the fourth-order Runge-Kutta method and Gill for predators and prey, one of which is in Figure 3 (b) overall almost the same estimation results. From the analysis it can be seen that the difference between the two methods is very small. Almost approaching zero. This is possible from the numerical solution between the fourth-order Runge-Kutta and Gill is almost the same, so there is no significant difference between the two methods.

From the analysis that has been done, the four parameters analyzed, namely the prey birth rate, predator death rate, decline in the number of prey populations and increase in the number of predator populations greatly affect the number of predator and prey populations. When the value of the increase in the number of predator populations is greater than the decrease in the number of prey populations with variations in the values $a > b$; $a = b$; and $a < b$ where a is the prey birth rate and b is the predator death rate, the prey and predator populations increase, then the prey population decreases while the predator population increases. This condition is stable and applies to all three conditions. This is because there are several influencing factors, such as environmental conditions (weather, temperature, etc.) and host plant conditions. However, from these graphs it can be seen that the two populations (predators and prey) influence and balance each other, namely when the prey population increases, the predator population also increases and vice versa. This happens because of the interaction of predation in the area.

Table 1. Computation time between RK4 and RKG methods

Runge-Kutta fourth order (s)	Runge-Kutta Gill (s)	Caption
1.0459	2.7494	Figure 3
1.0264	2.7972	Figure 4
1.0949	2.7791	Figure 5
1.1013	2.8467	Figure 6
1.0842	2.7725	Figure 7
1.1347	2.8181	Figure 8

Table 1 shows the computation time of the fourth-order Runge-Kutta and Gill methods. In the table above, for the time required in the iteration of the numerical solution of the Predator-Prey model, overall it can be seen that the fourth-order Runge-Kutta method is faster than the Gill Runge-Kutta method. The computation time obtained from the results of the numerical solution iteration cannot be used as a comparison of the two methods, because in solving the Predator-Prey model numerically, the formulations of the two methods are different. The computation time of the Gill Runge-Kutta is longer than the computation time of the fourth-order Runge-Kutta because the solutions are more than the fourth-order Runge-Kutta [39], [40]. In this case, the fourth-order Runge-Kutta and Gill methods as an alternative solution to the analytical method, both are good methods because from the explanation of the simulation results analysis above, it shows that the solutions produced by the fourth-order Runge-Kutta and Gill methods are close to the actual values so that it can be stated that the interaction between predators and prey influences and balances each other. However, there is no significant difference between these two methods in determining which method is better in solving the Predator-Prey model. This is because the Predator-Prey model cannot be solved analytically or does not have an exact solution, so the difference between the two methods cannot be seen from the analytical solution approach.

This study has several important implications, both in academic and practical contexts. In academic aspects, the results of the numerical analysis of the predator-prey model using the fourth-order Runge-Kutta and Gill methods can enrich the literature on the application of numerical methods in solving problems involving interactions between species [41], [42]. In addition, this study also contributes to the development of more accurate mathematical models to describe population dynamics in natural and artificial ecosystems. The application of these two numerical methods shows higher accuracy and can be used as a reference for further research that examines the predator-prey interaction model in more complex contexts, such as natural resource management or agricultural ecosystems.

Practically, the implications of this study are very relevant for ecosystem management and monitoring population balance in the environment. The predator-prey model solved by this numerical method can be used to design more effective management strategies in various sectors, including agriculture and nature conservation [43], [44]. For example, in agricultural pest management, this model can help formulate policies to maintain the balance between predators and prey, thereby reducing the use of hazardous pesticides. In addition, this research can also provide deeper insights into how external factors, such as conservation policies or protected area management, can affect the stability of predator and prey populations in an ecosystem.

The main limitation of this study lies in the use of the fourth-order Runge-Kutta and Gill methods to solve the Predator-Prey model, which although providing good accuracy, is still limited to a certain order. In addition, the model used in this study does not consider external factors or more complex parameter variations, which may occur in real-world applications, such as climate change or human intervention in the ecosystem. Therefore, further research is recommended to explore the use of higher-order numerical methods, such as the fifth-order Runge-Kutta method or more, to improve the accuracy of the calculation results. In addition, the use of more specific and representative data from the ecosystem or predator-prey population studied will provide a deeper and more realistic understanding of the dynamics of the population, as well as enrich the application of mathematical models in the context of natural resource management and conservation.

4. CONCLUSION

The conclusion that can be drawn from this study is that the fourth-order Runge-Kutta and Gill methods are both good methods. From the results of the analysis of the difference in estimates of the fourth-order Runge-Kutta and Gill for predators and prey, these two methods do not show significant differences in determining a better method in solving the Predator-Prey model. Because the Predator-Prey model cannot be solved analytically, so the difference between the two methods cannot be seen from the analytical solution approach. The simulation results using the fourth-order Runge-Kutta and Gill methods show that the greater the value of b, the prey population increases with a value of $\alpha > \beta$, and the smaller the values of α and β given, the interaction process between the two populations will slow down and the prey population will increase. This is because the prey population reproduces very quickly than the predator but there is no explosion between the prey and the predator in birth or death so that the interaction process is stable. In this study, the problem discussed is the numerical solution of the Predator-Prey model using the fourth-order Runge-Kutta and Gill methods, so that for subsequent studies it is recommended to use other higher-order numerical methods with more specific data.

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