Interval: Indonesian Journal of Mathematical Education
Vol. 2, No. 1, June 2024, pp. 78~89
ISSN: 3021-7857, DOI: 10.37251/ijome.v2i1.1359

Numerical Solution Analysis of Planetary Motion Models Using the Runge-Kutta Method

Ines Buissa Baluta[1], Halimah Tu'sadiyah [2], Makungu Bulayi[3], Talha İbtisam[4], Tharaka Jeewantha[5]
[3]Dapertment Natural Sciences, Mathematics Sciences, Universidade 11 de Novembro, Cabinda, Angola
[2] Mathematics Education Teacher, State Junior High School 1, Bengkulu, Indonesia
[3]Dapertment Natural Sciences, Mathematics Sciences, Ruaha Catholic University, Iringa, Tanzania
[4]Dapartment Natural Sciences, Mathematical Sciences, Université Kasdi Merbah Ouargla, Ouragla, Algeria
[5]Dapartment Natural Sciences, Mathematical Sciences, Bank of Ceylon, Colombo, Sri Lanka

Article Info

Article history:

Received Apr 15, 2024
Revised May 20, 2024
Accepted Jun 23, 2024
Online First Jun 26, 2024

Keywords:

Numerical Simulation
Planetary Motion Model
Planetary Orbit Profile
Runge-Kutta Fourth Order
Trajectory Stability Parameters

ABSTRACT

Purpose of the study: This study aims to solve the planetary motion model numerically using the fourth-order Runge-Kutta method and analyze the planetary motion profile through the resulting numerical solutions.

Methodology: The process is carried out by solving the planetary motion model numerically using the fourth-order Runge-Kutta method, creating a program from the numerical solution, and simulating the program with variations in the parameters of the stability of the trajectory and the distance of the planet to the sun. The simulation results are in the form of estimates of the speed of the planet's motion in the x and y directions against time, and the influence of these parameters on the trajectory and velocity graphs are analyzed.

Main Findings: Simulations show that the trajectory stability parameter and the planet's distance to the sun affect the planet's trajectory and velocity graphs. On the trajectory graph, the planet's distance to the sun affects the aphelion, minor axis, and major axis values of the orbit. The closer the planet is to the sun, the smaller its orbit, and vice versa. On the velocity graphs in the x and y directions, the best stability is obtained when the trajectory stability parameter is 2, in accordance with Newton's law of gravity.

Novelty/Originality of this study: The novelty of this research lies in the application of the fourth-order Runge-Kutta method to solve the planetary motion model numerically, without requiring function derivatives. This research also connects the numerical results with Newton's law of gravity to understand the relationship between the distance of a planet to the sun and its orbital pattern, thus enriching the numerical approach in astrophysical analysis.

Corresponding Author:
Ines Buissa Baluta,
Department of Natural Sciences, Faculty of Science and Mathematics, Universidade 11 de Novembro,
Cabinda Road, Angola, Cabinda, Angola
Email: inesbuissa@gmail.com

1. INTRODUCTION

A scientist named Johannes Kepler discovered the laws that govern the movement of the earth around the sun, especially concerning changes in the distance between the earth and the sun. Kepler's laws about the earth's orbit around the sun can be stated as follows: first, that the path of each planet in the solar system when orbiting the sun forms an ellipse. Second, that the radius vector will move to form the same area for every equal time. And third, that the time to orbit one period of rotation around the sun has a relationship to the semimajor axis of each planet which is constant in magnitude [1]-[3].

In modern times, Kepler's laws are used to approximate the orbits of satellites and objects orbiting the Sun, all of which were undiscovered at the time of Kepler's life (outer planets and asteroids). These laws are then applied to all small objects orbiting much larger objects, although some aspects such as atmospheric friction (motion in low orbits), or relativity and the presence of other objects can make the calculations inaccurate for various purposes [4]-[6].

Several studies have been conducted on the topic of planets orbiting the sunResearch examining the motion patterns of planets orbiting the sun. This research is more focused on creating (simulating) the motion patterns of planets orbiting the sun and is carried out using a simulation program with variations in the input values of the number of orbits and variations in the central radius between orbits [7]-[9]. research that has been done before also proves that Kepler's 2nd law on the path of each planet orbiting the sun is elliptical. And research related to the fourth-order Runge-Kutta method has also been done a lot, including research that compares the accuracy between the fourth-order Runge-Kutta method and the third-order Predictor-Corrector method. The results of the research are that the error value produced by the fourth-order Runge-Kutta method scheme is smaller than the third-order Predictor-Corrector scheme.. Thus, the Runge-Kutta method is more accurate than the Predictor-Corrector method [10]-[12].

Therefore, this study offers a novelty in the form of integrating the fourth-order Runge-Kutta method in the numerical analysis of the planetary motion model around the sun, not only to solve the equations of motion, but also to analyze the influence of trajectory stability parameters and the distance of the planet to the sun on the trajectory profile and the planet's velocity in the x and y directions. Different from previous studies, this study not only validates Newton's law of gravity or the elliptical trajectory pattern, but also identifies the optimal parameters for the stability of the planet's trajectory [13]-[15].

Thus, this research provides significant contribution in numerical modeling of astrophysics, especially in the solar system. The fourth-order Runge-Kutta method used can be a reference for further studies in simulating the trajectory of other celestial objects, such as comets or asteroids. In addition, the analysis of trajectory stability parameters can be applied in complex orbital mechanics systems, for example in designing artificial satellite paths [16], [17].

This study is important to bridge the gap in the numerical analysis of planetary motion models around the sun. Although previous studies have discussed the trajectory pattern and validation of the law of gravity, there has been no numerical approach that focuses on connecting the trajectory stability parameters with the accuracy of the planetary motion graph. This study is also relevant to improving the accuracy of solutions in modern astrophysical analysis. Therefore, the author is interested in analyzing the numerical solution of planetary motion using the fourth-order Runge-Kutta method [18]-[20].

The purpose of this study is to solve the planetary motion model numerically using the fourth-order Runge-Kutta method, which is known as a numerical method with a high degree of accuracy without requiring function derivatives. This study also aims to analyze the effect of trajectory stability parameters and planetary distance on the trajectory profile and the planetary motion velocity in the xx and yy directions. Through this approach, the study is expected to provide a deeper understanding of the planetary motion pattern in orbiting the sun, both in terms of orbital trajectory and the stability of its motion velocity. In addition, this study aims to provide simulations that are able to validate Newton's law of gravity numerically, with results that are in accordance with and support classical physics theory. Thus, this study not only contributes to the development of more accurate numerical models, but also broadens insight into the stability of planetary trajectories, which can be the basis for further astrophysical studies [21]-[23].

2. RESEARCH METHOD

The steps to solve this thesis problem are to first study the literature on the planetary motion model [24-[26]. Then, numerically solve the planetary motion model using the fourth-order Runge-Kutta Method. After getting a numerical solution, the next step is to create a program from the numerical solution that has been obtained [27]-[29]. Next, simulate the program by varying the parameter values. And the last step is to analyze the simulation results. Systematically, these steps can be seen in the following chart:

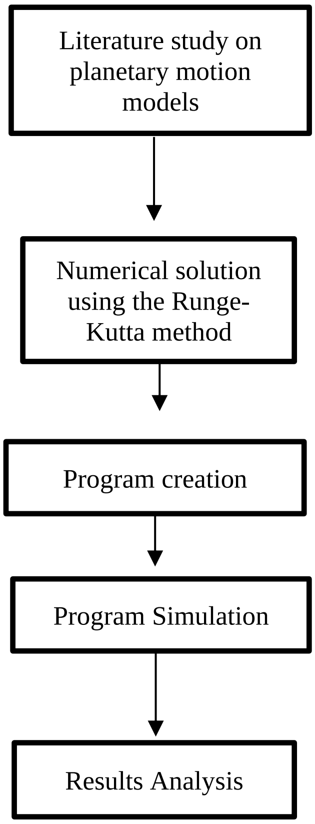

Chart 1. Steps to analyze simulation results

a. Literature study of planetary motion models

The planetary motion model used in this thesis is in the form of a function equation for the speed of a planet in orbiting the sun which is the result of Supardi's research (No Year) in his thesis. Viewed from the dependent variable, First order linear ordinary differential equation. So that the equation can be solved numerically by the fourth order Runge-Kuta Method [30]-[32].

b. Numerically with the fourt-order Runge-Kutta method

The equation used in this study is the fourth-order Runge-Kutta equation. In this process, the fourth-order Runge-Kutta equation is sought for its numerical solution using the fourth-order Rungekutta method and defining the variables $k1$, $k2$, $k3$ and $k4$ according to the general scheme [33]-[35].

c. Program Creaction

The software that will be used in creating this program is Matlab R2009a software. The procedure for making a planetary motion model analysis program is as follows:

1) Determination of parameter values

The determination of these parameters is taken from several literatures related to the movement of planets around the sun. The parameter values include the initial position x, y, the universal gravitational constant G, the mass of the planet m, the distance of the planet to the sun r, the eccentricity value e and the stability of the planet's trajectory β.

2) Process

The process referred to here is creating a program for the fourth-order Runge-Kutta method.

3) Output

The output that will be produced from this program is a graph of the planet's motion profile and a graph of the velocity function of the direction x and y against time.

d. Program Simulation

After the program is complete, the next step is to conduct a simulation by varying the parameters that affect the motion of the planet. In this simulation, there are several parameters that will be varied, namely the initial position x, y, the universal gravitational constant G, the mass of the planet m, the distance of the planet to the sun r, the eccentricity value e and the stability of the planet's trajectory β. Furthermore, the results of the simulation will be visualized in two dimensions [36]-[38].

e. Results Analysis

In this step, an analysis of the results obtained from the program simulation will be carried out. The analysis is carried out by considering the influence of parameters on the graphs produced by the program that have been produced in the previous step. This analysis aims to determine the profile of planetary motion [39]-[41].

3. RESULTS AND DISCUSSION

3.1. Numerical Solution

For directions x, (1)

$$Vx_{i+1} = Vx_i + \frac{1}{6}(k_1 + 2_{k2} + 2_{k3} + k_4)\Delta t \quad \ldots(1)$$
$$Vx_{i+1} = x_i - v_{xi+1}\Delta t$$

With,

$$k_{\mathbf{1}} = -\frac{GMx}{r^{\beta}}$$
$$k_{\mathbf{2}} = (-\frac{GMx}{r^{\beta}} - 0{,}5k_{\mathbf{1}}\Delta t)$$
$$k_{\mathbf{3}} = (-\frac{GMx}{r^{\beta}} - 0{,}5k_{\mathbf{2}}\Delta t)$$
$$k_{\mathbf{3}} = (-\frac{GMx}{r^{\beta}} - 0{,}5k_{\mathbf{3}}\Delta t)$$

As for the direction y,

$$Vx_{i+1} = Vx_i + \frac{1}{6}(k_1 + 2_{k2} + 2_{k3} + k_4)\Delta t \quad \ldots(2)$$
$$y_{i+1} = y_i - v_{yi+1}\Delta t$$
$$k_{\mathbf{1}} = -\frac{GMx}{r^{\beta}}$$
$$k_{\mathbf{2}} = (-\frac{GMy}{r^{\beta}} - 0{,}5k_{\mathbf{1}}\Delta t$$
$$k_{\mathbf{3}} = (-\frac{GMy}{r^{\beta}} - 0{,}5k_{\mathbf{2}}\Delta t$$
$$k_{\mathbf{4}} = (-\frac{GMx}{r^{\beta}} - k_{\mathbf{3}}\Delta t$$

From the equation (1) and (2) Next, the following planetary motion profile simulation is carried out.

3.2. Simulation and Program Analysis

The program used in this study is obtained from equations (1) and (2) which will produce 2 graphs, including the planetary motion profile graph and the velocity function graph of the direction x and y. The procedure for using the planetary motion profile program is to input the parameters of the planet's distance to the sun r and the eccentricity value e which produces program results in the form of perihelion, aphelion, minor axis and major axis values. These values are visualized in the form of a planetary motion trajectory with the point in the middle interpreted as the sun.

For the procedure of using the program of velocity function graph in the direction of x and y, the initial position parameters x, y, the universal gravitational constant G, the mass of the planet m, the distance of the planet to the sun r, the stability of the trajectory β and the speed of the planet in the direction of x, y are inputted. This program produces the results of the program in the form of planetary velocity values in the direction of x, y. These values are visualized in the form of an oscillation graph of the velocity function in the direction of x and y.

In this simulation, the data that the author uses is data that is expected to answer the problems in this thesis, namely to determine the profile of planetary motion. The planetary data in question is data that is in the ring in the Milky Way galaxy in the solar system, namely Mercury, Venus and Earth. The selection of this planetary data is based on the fact that this study pays more attention to how much influence the planetary trajectory stability parameter (β) has, so this data has answered the problems in this thesis.

Some of the parameters that are varied are the initial position x, y, the universal gravitational constant G, the mass of the planet m, the distance of the planet to the sun r, the eccentricity value e and the stability of the trajectory β. Some of the parameters that will be input are presented in the following table 1:

Table 1. Data on planets in the rings of the solar system

Planet	Distance of planets to the sun (r) × 10^9 m	Gravitational constant (G) × 10^{-11} m^3/s^2 . kg	Planet Massa (M) × 10^{24} Kg	Eccentricity value (e)
Mercury	57.9	6.67	3.289	0.206
Venus	108.2	6.67	48.737	0.007
Earth	159.6	6.67	5.98	0.017

Source: Halliday & Resnick[42]

The results of the program simulation with the data in table 4.1 are shown in figures 1 to 9 below:

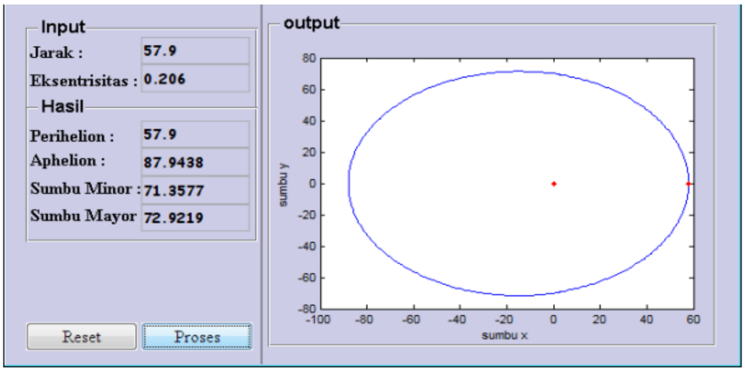

Figure 1. Mercury's transit chart

Figure 1 explains the graph of the trajectory of the planet Mercury with the given input values including the distance of the planet to the sun (r) = 57.9 × 10^9 m and the eccentricity value = 0.206. From the program simulation, the perihelion (closest point to the sun) = 57.9 × 10^9 m, aphelion (farthest point from the sun) = 87.9438 × 109 m, the minor axis of the planet Mercury = 71.3577 × 10^9 m and the major axis of the planet Mercury = 72.9219 × 10^9 m

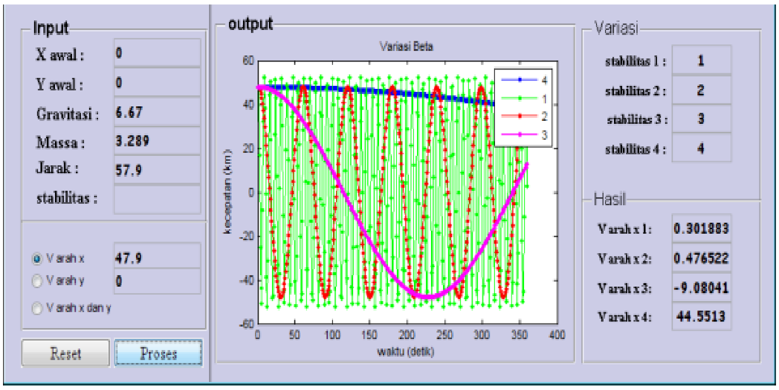

Figure 2. Graph of the velocity function of the planet Mercury in the x direction against time with variations in beta (β)

Figure 2 shows the graph of the velocity function of the planet Mercury in the x direction against time with the input value of the gravitational constant = 6.67×10^{-11} m^3/s^2 .kg, planet mass = 3.289 × 10^{24} kg, distance from the sun to the planet = 57.9 × 10^9 m, with stability variations (β_1) = 1, stability (β_2) = 2, , stability (β_3) = 3, stability variation (β_4) = 4 and the initial velocity of Mercury $v_0 x$ = 47.9 km/ s produces the value of the planet's velocity in the x direction according to the stability variation (β) yakni v_{x1} = 0.301883 $\frac{km}{s}$, v_{x2} = 0.476522 $\frac{km}{s}$, v_{x3} = -9.08041 $\frac{km}{s}$ dan v_{x4} = 44.5513 $\frac{km}{s}$. From the graph and the speed values produced by the simulation program above, it can be interpreted that the graph of the speed function of the planet Mercury in the x direction according to Newton's law of gravity is the variation value $(\beta_2) = 2$.

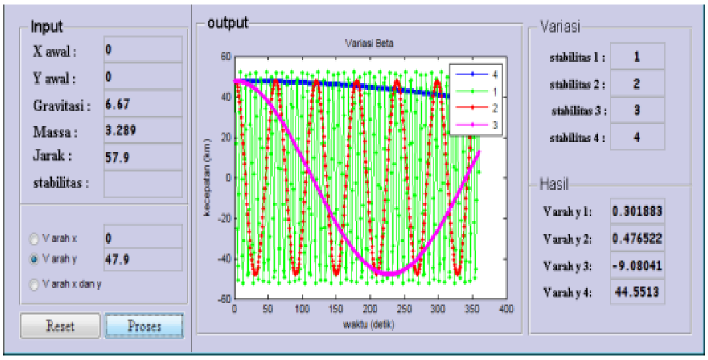

Figure 3. Graph of the velocity function of the planet Mercury in the y direction against time with beta variations. (β)

Figure 3 shows the graph of the velocity function of the planet Mercury in the y direction against time with the input value of the gravitational constant.= 6.67×10^{-11} m ^{3} /s^2 . kg , planet mass = 3.289×10^{24} kg, distance from the Sun to the planet = 57.9×10^9 m, with stability variations $(\beta 1) = 1$, stability $(\beta 2) = 2$, stability $(\beta 3) = 3$, stability variation $(\beta 4) = 4$ and the initial velocity of mercury $v_0 y = 47.9$ km/s produces the value of the planet's velocity in the y direction according to the stability variation (β) yakni $v_{y1} = 0.301883 \frac{km}{s}$, $v_{y2} = 0.476522 \frac{km}{s}$, $v_{y3} = -9.08041 \frac{km}{s}$dan $v_{y4} = 44.5513 \frac{km}{s}$. From the graph and speed values produced by the simulation program above, it can be interpreted that the graph of the speed function of the planet Mercury in the y direction is in accordance with Newton's law of gravitasi adalah nilai variasi $(\beta 2) = 2$.

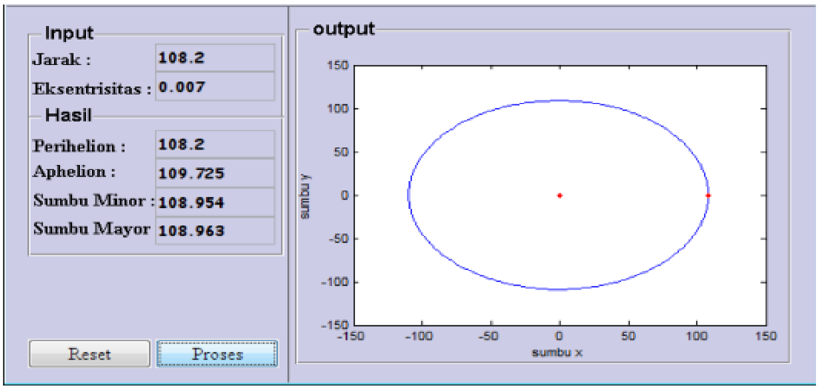

Figure 4. Graph of the path of the planet Venus

Figure 4 explains the graph of the path of the planet Venus with the given input values including the distance of the planet to the sun (r) = 108.2×10^9 m and the eccentricity value = 0.007. From the program simulation it was obtained *perihelion* (closest point to the sun) = 108.2×10^9 m, *aphelion* (furthest point from the sun) = 109.725×10^9 m, planetary minor axis Venus = 108.954×10^9 m and the major axis of the planet Venus = 108.963×10^9 m.

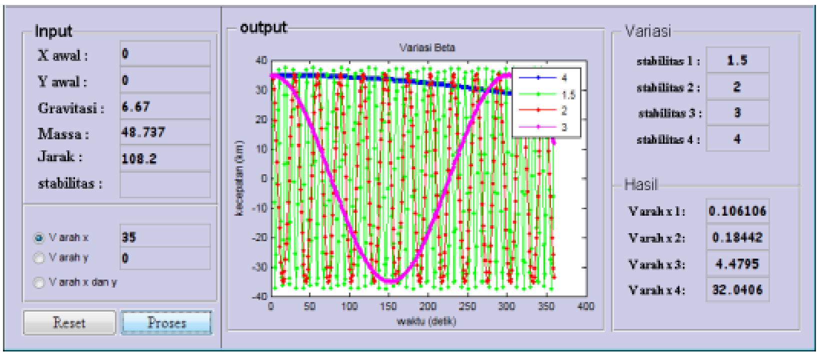

Figure 5. Graph of the velocity function of the planet Venus in the x direction against time with beta variations. (β)

Figure 5 explains the graph of the velocity function of the planet Venus in the x direction against time with the input value of the gravitational constant = 6.67×10^{-11} m^3/s^2 .kg, planet mass = 48.737×10^{24} kg, distance from the sun to the planet = $108{,}2 \times 10^9$ m, with stability variations (β_1) = 1,5, stability (β_2) = 2, s stability (β_3) = 3, stability variation (β_4) = 4 and initial velocity $v_0 x$ = 35 km/s produces the value of the planet's velocity in the direction x according to with stability variations (β) yakni v_{x1} = 0.106106 $\frac{km}{s}$, v_{x2} = 0.18442 $\frac{km}{s}$, v_{x3} = 4.4795 $\frac{km}{s}$ dan v_{x4} = 32.0406 $\frac{km}{s}$. From the graph and the speed values produced by the simulation program above, it can be interpreted that the graph of the speed function of the planet Venus in the x direction according to Newton's law of gravity is the value of the stability variation ($\beta 1$) = 2.

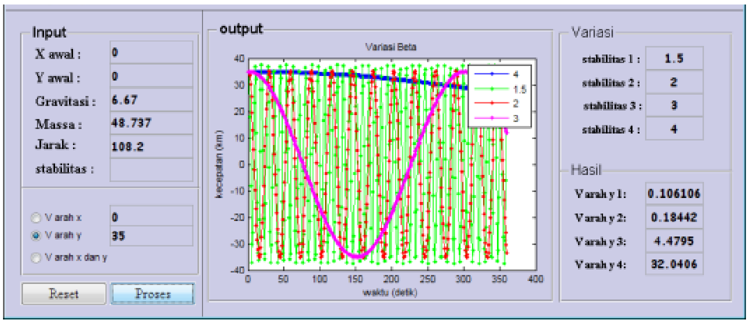

Figure 6. Graph of the velocity function of the planet Venus in the y direction against time with variations in beta. (β)

Figure 6 explains the graph of the velocity function of the planet Venus in the y direction against time with the input value of the gravitational constant = 6.67×10^{-11} m^3/s^2 . kg, mass of the planet = 48.737×10^{24} kg, distance from the sun to the planet = $108{,}2 \times 10^9$ m, with stability variations (β_1) = 1,5, stability (β_2) = 2, stability (β_3) = 3, stability variation (β_4) = 4 and initial velocity $v_0 y$ = 35 km/s produces the value of the planet's velocity in the y direction according to the stability variation (β) namely v_{y1} = 0.106106 $\frac{km}{s}$, v_{y2} = 0.18442 $\frac{km}{s}$, v_{y3} = 4.4795 $\frac{km}{s}$ and v_{y4} = 32.0406 $\frac{km}{s}$. From the graph and the speed values produced by the simulation program above, it can be interpreted that the graph of the speed function of the planet Venus in the y direction according to Newton's law of gravity is the value of the stability variation (β_1) = 2.

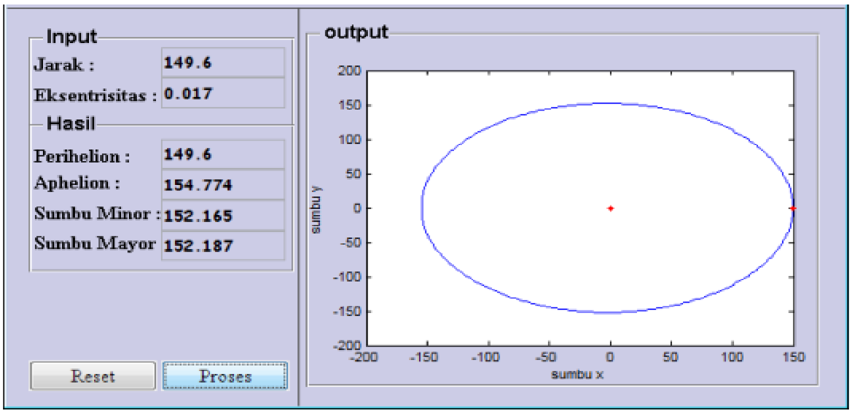

Figure 7. Graph of the Earth's orbit

Figure 7 explains the graph of the Earth's trajectory with the given input values including the distance of the planet to the sun (r) = 149.6×10^9 m and eccentricity value = 0.017. From the program simulation, the *perihelion* (closest point to the sun) was obtained. = 149.6×10^9 m, *aphelion* (furthest point from the sun) = 154.774×10^9 m, Earth's minor axis = 152.165×10^9 m and the major axis of the planet Earth = 152.187×10^9 m.

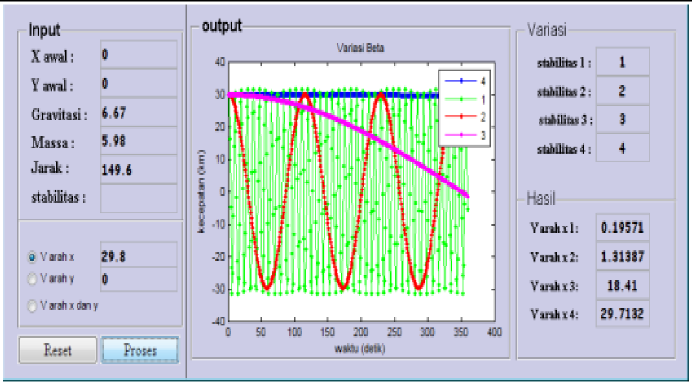

Figure 8. Graph of the Earth's velocity function in the x direction against time with beta variations (β)

Figure 8 explains the graph of the Earth's velocity function in the x direction against time with the input value of the gravitational constant = 6.67×10^{-11} m^3/s^2 .kg, planet mass = 5.98×10^{24} kg, distance from the sun to the planet = 149.6×10^9 m, with stability variations (β_1) = 1, stability (β_2) = 2, stability (β_3) = 3, stability variation (β_4) = 4 and initial velocity v_0x = 29.8 km/s produce the value of the planet's velocity direction x according to beta variation (β) namely $v_{x1} = 0.19571\ \frac{km}{s}$, $vx2 = 1.31387\ \frac{km}{s}$, $v_{x3} = 13.41\ \frac{km}{s}$dan $v_{x4} = 29.7132\ \frac{km}{s}$. From the graph and the speed values produced by the simulation program above, it can be interpreted that the graph of the speed function of planet Earth in the x direction according to Newton's law of gravity is the value of the stability variation (β_1) = 2.

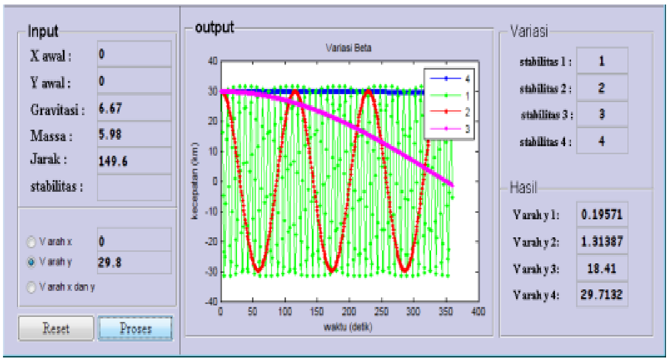

Figure 9. Graph of the Earth's velocity function in the y direction against time with variations in beta (β)

Figure 9 explains the graph of the Earth's velocity function in the y direction against time with the input value of the gravitational constant = 6.67×10^{-11} m^3/s^2 .kg, planet mass = 5.98×10^{24} kg, distance from the sun to the planet = 149.6×10^9 m, with stability variations (β_1) = 1, stability (β_2) = 2, stability (β_3) = 3, stability variation (β_4) = 4 and initial velocity v_0y = 29.8 km/s produces the value of the planet's velocity in the y direction according to the variation of beta (β) namely $v_{y1} = 0.19571\ \frac{km}{s}$, $v_{y2} = 1.31387\ \frac{km}{s}$, $v_{y3} = 13.41\ \frac{km}{s}$and $v_{y4} = 29.7132\ \frac{km}{s}$. From the graph and the speed values produced by the simulation program above, it can be interpreted that the graph of the speed function of planet Earth in the y direction according to Newton's law of gravity is the value of the stability variation (β_1) = 2.

3.3. Program Simulation Results

From the program simulation in Figures 1 to 9, the values related to the planetary motion profile and the value of the planetary velocity function in the x and y directions against time are obtained. In the planetary motion profile, the values obtained are as follows.

Table 2. Simulation results of the planetary motion profile program

Planet	Input		Output			
	$r \times 10^9$ m	e	$perihelion \times 10^9$ m	$Aphelion \times 10^9$ m	$Sumbu\ minor \times 10^9$ m	$Sumbu\ mayor \times 10^9$ m
Mercury	57.9	0.206	57.9	87.9438	71.3577	72.9219
Venus	108.2	0.007	108.2	109.725	108.954	108.963
Earth	149.6	0.017	149.6	154.774	152.165	152.165

Table 2 explains that the further the distance of the planet from the sun (r), the further the aphelion point. This means that the closer the distance between the planet and the sun, the closer the orbit/path of the planet in circling the sun. Likewise, the further the distance between a planet and the sun, the further the orbit/path of the planet in circling the sun. Meanwhile, in the planetary trajectory graph, the values obtained are as follows.

Table 3. Simulation results of the planetary velocity function program

Planet	Input					Output		
	G	M	r	v_{0x}	v_{0y}	β	v_x	v_y
Mercury	6.67	3.289	57.9	47.9	47.9	1	0.301883	0.301883
						2	0.476522	0.301883
						3	-9.08041	0.301883
						4	44.5513	0.301883
Venus	6.67	48.737	108.2	35	35	1.5	0.106106	0.106106
						2	0.18442	0.18442
						3	4.4795	4.4795
						4	32.0406	32.0406
Earth	6.67	5.98	149.6	29.8	29.8	1	0.19571	0.19571
						2	1.31387	1.31387
						3	18.41	18.41
						4	29.7132	29.7132

Table 3 shows that when beta β is equal to 1 and 1.5 the graph looks tight and unstable at the upper or lower limits. This explains that the planet orbits the sun too fast and is unstable in its path. While when β is equal to 3 and 4 the graph looks loose. This explains that the planet takes longer to orbit the sun than when β is equal to 2.

When β is equal to 2, then the planet orbits in its path in the most stable state compared to when β is equal to 1; 1.5; 3 or when β is equal to 4. This is in accordance with Newton's law of gravity which states that every particle of matter in the universe pulls on other particles with a force that is directly proportional to the product of the masses of the other particles and inversely proportional to the square of the distance separating them. Thus, in this study, Newton's law of gravity is proven to be true.

Previous research that has been conducted that has given rise to a gap with this research, shows fundamental differences in the focus of the methodology and approach used. This research uses the fourth-order Runge-Kutta method to solve the planetary motion model numerically. The main focus of this research is the simulation of the planetary orbit and the analysis of the effects of the parameters of the stability of the trajectory and the distance of the planet to the sun on the profile of the trajectory and the speed of the planet. On the other hand, previous research emphasizes the qualitative data analysis approach, including the method of collecting, compiling, and analyzing data systematically. This qualitative research has a broad scope, but is not directly related to astrophysical phenomena or numerical simulations[42]-[44].

The results and findings of both studies also show significant differences. Numerical research produces a graph of the planet's orbital trajectory and speed of motion that is varied based on the parameters of trajectory stability and distance to the sun. These findings support Newton's law of gravity and provide a clear visualization of the planet's orbital pattern. In contrast, qualitative research focuses on the process of processing data to find meaningful patterns and relationships, providing systematic methodological guidance, but without data-based visualization or simulation results[45]-[47].

Overall, the implementation gap between the two studies is clear. Numerical research is heavily focused on astrophysical applications, particularly planetary orbits, while qualitative research is methodological and broader in scope, but has no direct connection to numerical applications in the exact domain. This difference emphasizes that numerical research is more in-depth and specific to astrophysics, while qualitative research provides general guidance that can be applied to a variety of research fields.

This study offers a new approach by applying the fourth-order Runge-Kutta method to solve the planetary motion model numerically without requiring function derivatives. This study also successfully connects the numerical results with Newton's law of gravity, providing a deep understanding of the effects of the trajectory stability parameter (β) and the planet's distance from the sun on the orbital profile and planetary velocity. Not only does this study validate Newton's law of gravity, it also identifies the optimal value of the trajectory stability parameter (β = 2), which provides the most stable orbital trajectory[48]-[50].

This study has several limitations that need to be considered for future development. First, the analysis conducted only focuses on the parameters of trajectory stability (β) and the distance of the planet to the sun, without considering the influence of external disturbances such as gravity from other planets, interactions between celestial bodies, or relativity effects. This can limit the accuracy of the model in representing real conditions, especially in a complex solar system. Second, the data used in this study is limited to the planets in the inner solar system, namely Mercury, Venus, and Earth. Therefore, the results of the study may be less representative for analyzing the trajectories of other celestial objects, such as planets in the outer solar system, asteroids, or comets, which have more complex characteristics and dynamics.

Further research is recommended to expand the model parameters by including variables such as changes in solar mass and gravitational effects from other planets to improve accuracy and representation of real conditions. In addition, the model can be tested on other celestial objects such as asteroids, comets, or satellites to understand trajectories under more complex conditions. Integration with artificial intelligence (AI) is also recommended to automate the optimization of trajectory stability parameters, resulting in more efficient and accurate solutions. The use of multiplatform platforms such as Python or C++ is expected to overcome the limitations of MATLAB, allowing simulations on a larger scale and in a variety of environments. This approach will strengthen the generalization of the model and expand its applications in modern astrophysics.

4. CONCLUSION

Based on the research results, several important conclusions were obtained regarding the planetary motion model and the influence of parameters on the stability of the planet's velocity in orbiting the sun. First, the solution of the planetary motion model in the form of a velocity function equation can be done using the fourth-order Runge-Kutta method. This method is a one-step method that provides a high level of accuracy without requiring function derivatives, so that the results obtained can be used as a basis for analyzing the influence of certain parameters, especially the parameter β. Second, the parameter β is proven to be the most influential factor on the planet's velocity stability graph. The appropriate and stable value of the parameter β for the planetary motion model is 2, consistent with Newton's law of gravity. Third, the distance between the planet and the sun affects the aphelion point and the planet's orbit. The closer the planet is to the sun, the smaller its orbital path, while the further the distance, the larger its orbital path. This shows a close relationship between the distance of the planet from the sun and its stability and trajectory pattern.

The results of this study have significant implications in various fields. In astrophysical modeling, this study can be the basis for the development of simulations of the trajectories of other celestial objects, such as asteroids or comets, with a high degree of accuracy. These findings are also useful in the design of orbital systems, where simulation results related to trajectory stability parameters can be applied to design artificial satellite paths, ensuring orbital stability in complex orbital mechanics systems. In addition, this study provides important contributions to education and research, especially as teaching materials and references for further studies in the fields of astrophysics and numerical modeling. This is expected to improve students' and researchers' understanding of planetary orbit phenomena and broaden their horizons in the application of modern astrophysics concepts.

ACKNOWLEDGEMENTS

The author would like to thank all parties who have contributed to the continuation of this research, so that this research can be completed. The author hopes that this research can provide a good impact to those in need.

REFERENCES

[1] C. W. Sitorus and Y. Yahfizham, “Systematic literature review: Analisis kemampuan berpikir komputasi siswa menggunakan software matematika geogebra,” *Pendek. J. Pendidik. Berkarakter*, vol. 2, no. 3, pp. 107–116, 2024, doi: 10.51903/pendekar.v2i3.736.

[2] A. Sugandha, “Review persamaan black-Scholes fraksional dimodifikasi,” *Perwira J. Sci. Eng.*, vol. 1, no. 2, pp. 26–37, 2022, doi: 10.54199/pjse.v1i2.68.

[3] D. U. Iswavigra and L. Endriani Zen, “Systematic literature review: pengaplikasian metode VIKOR dalam decision upport system,” *J. Inf. dan Teknol.*, vol. 5, no. 3, pp. 13–19, 2023, doi: 10.60083/jidt.v5i3.379.

[4] S. Solihun, N. I. Kadek, and P. Astuti, “Kajian penggunaan matematika dalam bidang perikanan,” *GANEC SWARA Media Inf. Ilm. Univ. Mahasaraswati Mataram*, vol. 17, no. 2, pp. 578–586, 2023, doi: 10.35327/gara.v17i2.459.

[5] Y. Trisnawati, “Hipnoterapi untuk menurunkan nyeri dismenore:tinjauan literatur,” *PREPOTIF J. Kesehat. Masy.*, vol. 7, no. 1, p. 640, 2023, doi: 10.31004/prepotif.v7i1.12033.

[6] M. Ciarniello *et al.*, “The origin and composition of saturn’s ring moons,” *Space Sci. Rev.*, vol. 220, no. 7, 2024, doi: 10.1007/s11214-024-01103-z.

[7] A. Sciola, F. Toffoletto, D. Alexander, K. Sorathia, V. Merkin, and A. Farrish, “Incorporating inner magnetosphere current-driven electron acceleration in numerical simulations of exoplanet radio emission,” *Astrophys. J.*, vol. 914, no. 1, p. 60, 2021, doi: 10.3847/1538-4357/abefd9.

[8] J. G. A. Guzmán, V. Florinski, G. Tóth, S. Sharma, B. van der Holst, and M. Opher, “Numerical modeling of energetic charged-particle transport with SPECTRUM Software: general approach and artificial effects due to eield discretization,” *Astrophys. J. Suppl. Ser.*, vol. 272, no. 2, p. 46, 2024, doi: 10.3847/1538-4365/ad4637.

[9] S. M. Shaaban, R. A. Lopez, M. Lazar, and S. Poedts, “Numerical simulations of temperature anisotropy instabilities

stimulated by suprathermal protons," *Astron. Astrophys.*, vol. 86, pp. 1–11, 2024, doi: 10.1051/0004-6361/202348780.
[10] P. A. Akimov, M. L. Mozgaleva, and T. B. Kaytukov, "Numerical solution of the problem of beam analysis with the use of b-spline finite element method," *Int. J. Comput. Civ. Struct. Eng.*, vol. 16, no. 3, pp. 12–22, 2020, doi: 10.22337/2587-9618-2020-16-3-12-22.
[11] T. E. Khosrow Maleknejad, Jalil Rashidinia, "Numerical solutions of distributed order fractional differential equations in the time domain using the Müntz–Legendre wavelets approach," *Numer. Methods Partial Differ. Equ.*, vol. 37, no. 1, pp. 707–731, 2020, doi: 10.1002/num.22548.
[12] A. Bahatmaka *et al.*, "Analytical review ofn numerical analysis in Hydrodynamic performance of the ship: Effect to hull-form modifications," *Mek. Maj. Ilm. Mek.*, vol. 23, no. 1, p. 54, 2024, doi: 10.20961/mekanika.v23i1.83635.
[13] J. W. Lee, K. J. Jung, and Y. J. Kim, "Numerical analysis on performance of induced gas flotation machine using MUSIG model," *Eng. Appl. Comput. Fluid Mech.*, vol. 14, no. 1, pp. 778–789, 2020, doi: 10.1080/19942060.2020.1771426.
[14] J. G. O'Rourke *et al.*, "Venus, the Planet: Introduction to the evolution of earth's sister planet," *Space Sci. Rev.*, vol. 219, no. 1, 2023, doi: 10.1007/s11214-023-00956-0.
[15] M. Kuman and M. Kuman, "Why are the axes of all atars in The milky way tilted and wobbling ? (In torus representation)," vol. 3, no. 2021, pp. 268–270, 2024, doi: 10.31.08.2024.
[16] V. H. Hansteen, J. Martinez-Sykora, M. Carlsson, B. De Pontieu, M. Gošić, and S. Bose, "Numerical simulations and observations of Mg ii in the solar chromosphere," *Astrophys. J.*, vol. 944, no. 2, p. 131, 2023, doi: 10.3847/1538-4357/acb33c.
[17] M. C. Mignone, Andrea, Gabriele Inghirami b, Francesco Rubini, Raniero Cazzaniga and M. Rosa-Clot, "Numerical simulations of wind-loaded floating solar panels," *Sol. Energy*, vol. 219, pp. 42–29, 2021, doi: 10.1016/j.solener.2020.11.079 Get rights and content.
[18] D. Breuer, T. Spohn, T. Van Hoolst, W. van Westrenen, S. Stanley, and N. Rambaux, *Interiors of earth-like Planets and satellites of the solar system*, vol. 43, no. 1. Springer Netherlands, 2022. doi: 10.1007/s10712-021-09677-x.
[19] Z. Shareefdeen and H. Al-Najjar, "Pollution effects and management of orbital space debris," *ACS Omega*, vol. 9, no. 5, pp. 5127–5141, 2024, doi: 10.1021/acsomega.3c06887.
[20] B. C. Yalçın, C. Martinez, M. Hubert Delisle, G. Rodriguez, J. Zheng, and M. Olivares-Mendez, "ET-Class: An energy transfer-based classification of Space debris removal methods and missions," *Front. Sp. Technol.*, vol. 3, no. March, pp. 1–23, 2022, doi: 10.3389/frspt.2022.792944.
[21] M. V. H. Khan and E. L. Ntantis, "Space Debris: Overview and mitigation strategies," *Proc. 8th Int. Conf. Res. Technol. Educ. Space, H-sp.*, vol. 8, pp. 25–26, 2021, doi: 10.3389/frobt.2021.686723.
[22] R. Diehl *et al.*, "The radioactive nuclei and in the Cosmos and in the solar system," *Publ. Astron. Soc. Aust.*, vol. 38, 2021, doi: 10.1017/pasa.2021.48.
[23] E. Fiandrini *et al.*, "Numerical modeling of cosmic rays in the heliosphere: Analysis of proton data from AMS-02 and PAMELA," *Phys. Rev. D*, vol. 104, no. 2, p. 23012, 2021, doi: 10.1103/PhysRevD.104.023012.
[24] B. Liu and J. Ji, "A tale of planet formation: From dust to planets," *Res. Astron. Astrophys.*, vol. 20, no. 10, 2020, doi: 10.1088/1674-4527/20/10/164.
[25] A. Emsenhuber, C. Mordasini, R. Burn, Y. Alibert, W. Benz, and E. Asphaug, "The new generation planetary population synthesis (NGPPS)," *Astron. Astrophys.*, vol. 656, p. A70, 2021, doi: 10.1051/0004-6361/202038863.
[26] D. Turrini *et al.*, "Tracing the formation history of giant planets in protoplanetary disks with carbon, oxygen, nitrogen, and sulfur," *Astrophys. J.*, vol. 909, no. 1, p. 40, 2021, doi: 10.3847/1538-4357/abd6e5.
[27] L. Ma and J. Fuller, "Orbital Decay of Short-period Exoplanets via Tidal Resonance Locking," *Astrophys. J.*, vol. 918, no. 1, p. 16, 2021, doi: 10.3847/1538-4357/ac088e.
[28] Y.-D. Tsai, D. Farnocchia, M. Micheli, S. Vagnozzi, and L. Visinelli, "Constraints on fifth forces and ultralight dark matter from OSIRIS-REx target asteroid Bennu," *Commun. Phys.*, vol. 7, no. 1, pp. 1–7, 2023, doi: 10.1038/s42005-024-01779-3.
[29] A. B. T. Penzlin, W. Kley, and R. P. Nelson, "Parking planets in circumbinary discs," *Astron. Astrophys.*, vol. 645, p. A68, 2021, doi: 10.1051/0004-6361/202039319.
[30] H. Rahnejat, P. M. Johns-Rahnejat, N. Dolatabadi, and R. Rahmani, "Multi-body dynamics in vehicle engineering," *Proc. Inst. Mech. Eng. Part K J. Multi-body Dyn.*, vol. 238, no. 1, pp. 3–25, 2024, doi: 10.1177/14644193231181666.
[31] M. H. Santoso, "Application of association rule method using apriori algorithm to find sales patterns case study of Indomaret Tanjung Anom," *Brill. Res. Artif. Intell.*, vol. 1, no. 2, pp. 54–66, 2021, doi: 10.47709/brilliance.v1i2.1228.
[32] D. Vinsensia, S. Amri, J. Sihotang, and H. T. Sihotang, "New method for identification and response to infectious disease patterns based on comprehensive health service data," *MATRIK J. Manajemen, Tek. Inform. dan Rekayasa Komput.*, vol. 23, no. 3, pp. 583–592, 2024, doi: 10.30812/matrik.v23i3.4000.
[33] A. G. Gad, *Particle swarm optimization algorithm and its applications: A systematic review*, vol. 29, no. 5. Springer Netherlands, 2022. doi: 10.1007/s11831-021-09694-4.
[34] J. Alman and V. V. Williams, "A refined laser method and faster matrix multiplication," *Proc. Annu. ACM-SIAM Symp. Discret. Algorithms*, vol. 3, no. 21, pp. 522–539, 2021, doi: 10.46298/theoretics.24.21.
[35] M. M. Rahman and Y. Watanobe, "ChatGPT for education and research: Opportunities, threats, and strategies," *Appl. Sci.*, vol. 13, no. 9, 2023, doi: 10.3390/app13095783.
[36] C. Wang, B. Amon, K. Schulz, and B. Mehdi, "Factors that influence nitrous oxide emissions from agricultural soils as well as their representation in simulation models: A review," *Agronomy*, vol. 11, no. 4, 2021, doi: 10.3390/agronomy11040770.
[37] X. Wang, D. Chen, G. Pang, S. A. Anwar, T. Ou, and M. Yang, "Effects of cumulus parameterization and land-surface hydrology schemes on tibetan plateau climate simulation during the wet season: insights from the RegCM4 model," *Clim. Dyn.*, vol. 57, no. 7–8, pp. 1853–1879, 2021, doi: 10.1007/s00382-021-05781-1.
[38] V. S. Erturk, E. Godwe, D. Baleanu, P. Kumar, J. Asad, and A. Jajarmi, "Novel fractional-order Lagrangian to describe

motion of beam on nanowire," *Acta Phys. Pol. A*, vol. 143, no. 3, pp. 265–272, 2021, doi: 10.12693/APhysPolA.140.265.
[39] R. B. Kristiawan, F. Imaduddin, D. Ariawan, Ubaidillah, and Z. Arifin, "A review on the fused deposition modeling (FDM) 3D printing: Filament processing, materials, and printing parameters," *Open Eng.*, vol. 11, no. 1, pp. 639–649, 2021, doi: 10.1515/eng-2021-0063.
[40] M. Singh, A. K. Jakhar, and S. Pandey, "Sentiment analysis on the impact of coronavirus in social life using the BERT model," *Soc. Netw. Anal. Min.*, vol. 11, no. 1, pp. 1–11, 2021, doi: 10.1007/s13278-021-00737-z.
[41] J. Płotka-Wasylka and W. Wojnowski, "Complementary green analytical procedure index (ComplexGAPI) and software," *Green Chem.*, vol. 23, no. 21, pp. 8657–8665, 2021, doi: 10.1039/d1gc02318g.
[42] H. & Resnick, *Foundamnetals of Physics*. New York, 1986.
[43] S. J. Walters, R. J. Turner, and L. K. Forbes, "a Comparison of explicit Runge-Kutta methods," *ANZIAM J.*, vol. 64, no. 3, pp. 227–249, 2022, doi: 10.1017/S1446181122000141.
[44] K. Aliyi Koroche, "Numerical solution of first order ordinary differential equation by using runge-kutta method," *Int. J. Syst. Sci. Appl. Math.*, vol. 6, no. 1, p. 1, 2021, doi: 10.11648/j.ijssam.20210601.11.
[45] T. Umeda and R. Ozaki, "Advanced numerical techniques for time integration of relativistic equations of motion for charged particles," *Earth, Planets Sp.*, vol. 75, no. 1, 2023, doi: 10.1186/s40623-023-01902-8.
[46] Y. Yang, J. Yan, N. Jian, K. Matsumoto, and J. P. Barriot, "Numerical model of Phobos' motion incorporating the effects of free rotation," *Astron. Astrophys.*, vol. 685, 2024, doi: 10.1051/0004-6361/202039412.
[47] S. N. Ordinary and D. Equations, "Formulative Visualization of Numerical Methods for Solving Non-Linear Ordinary Numbers Equations Formulative Visualization of Differential Numerical Methods for Solving Non-Linear Ordinary Differential Equations Header Even Pages Header Odd Pages Give Pag," *Nepal J. Math. Sci.*, vol. 2, no. 2, pp. 79–88, 2021, doi: 10.3126/njmathsci.v2i2.40126.
[48] R. D. Santiago Acosta, E. M. Hernández Cooper, A. Hernández Medina, and R. Gómez Castillo, "Interactive mechanical systems using mathematica," *J. Phys. Conf. Ser.*, vol. 1936, no. 1, 2021, doi: 10.1088/1742-6596/1936/1/012017.
[49] W. Tan, J. Wu, Z. Liu, X. Wu, and J. Zhang, "Research on nonlinear dynamic characteristics of high-speed gear in two-speed transmission system," *Sci. Rep.*, vol. 13, no. 1, pp. 1–10, 2023, doi: 10.1038/s41598-023-47981-1.
[50] A. I. Ismail and H. A. Ghulman, "New stabilization properties of pendulum models applying a large parameter," *Complexity*, vol. 2022, 2022, doi: 10.1155/2022/2704012.
[51] S. Nasir, A. S. Berrouk, T. Gul, I. Zari, W. Alghamdi, and I. Ali, "Unsteady mix convectional stagnation point flow of nanofluid over a movable electro-magnetohydrodynamics Riga plate numerical approach," *Sci. Rep.*, vol. 13, no. 1, pp. 1–12, 2023, doi: 10.1038/s41598-023-37575-2.