



The Role of Analyze-Identify-Connect in Enhancing Students' Conceptual Understanding in Mathematics: A Quasi-Experimental Approach

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ABSTRACT

Purpose of the study: The study aimed to compare the effects of AIC learning model and conventional method of teaching on students' conceptual understanding in mathematics.

Methodology: The study utilized a quasi-experimental design, with data on students' conceptual understanding collected by a researcher-developed test aligned to NCTM standards. Data analysis was conducted using SPSS with ethical review and informed consent procedures in place.

Main Findings: The findings demonstrate that the AIC learning model significantly enhances students' conceptual understanding in geometry, as evidenced by higher N-gain scores and a substantial effect size of 0.83 in the medium range. The experimental group outperformed the control group, shifting learning outcomes into a more effective range for conceptual development. This structured approach facilitates meaningful cognitive restructuring, marking a notable scientific contribution to mathematics education research.

Novelty/Originality of this study: This study introduces the AIC learning model as a pedagogical framework that systematically promotes conceptual understanding and student engagement in mathematics classrooms. Theoretically, it advances instructional design by demonstrating how structured, interactive methods can facilitate cognitive restructuring and deeper learning processes. These contributions provide a foundation for further research on the sustained effect of such approaches and offer actionable guidance for educators seeking to enhance mathematics teaching.

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1. INTRODUCTION

A weak foundation in mathematical concepts, particularly in geometry, is evident from students' performance in solving mathematical problems. In the Philippine context, recent local and international assessments (PISA 2022, TIMSS 2019) and national tests (NAT 2022-2023) confirm that Filipino students consistently perform poorly in mathematics, reading, and science. Several topics in grade nine junior high school geometry have been identified as the least learned competencies or the least mastered topics in the subject (DepED, 2025). Poor understanding of geometric definitions and concepts has significant implications for future mastery, often resulting in difficulty and failure [1], [2]. Research indicates that students frequently lack background knowledge about angles, measurement, and shapes, leading to errors in reasoning and basic

operations [3]. These findings underscore the need for students to develop a stronger understanding of geometry concepts and related skills.

Multiple interrelated factors contribute to students' weak conceptual foundations in mathematics. These challenges are frequently attributed to cognitive and pedagogical factors [4]. Research consistently demonstrates that students with weak foundational skills struggle with mathematics and continue to fall behind in advanced topics [5]. A lack of understanding of mathematical concepts and operations impedes problem-solving abilities, often resulting in poor performance [6]. Insufficient conceptual understanding also limits students' capacity to apply knowledge to novel mathematical problems [7]. These findings suggest a strong link between students' mathematical performance and their conceptual understanding.

Teaching methods represent a significant factor contributing to students' poor performance in mathematics [8], [9]. The choice of instructional approach is critical for fostering an engaging and supportive learning environment that promotes student success [10]. Conventional teaching methods, which emphasize content delivery and procedural skills, have been criticized for failing to cultivate deeper conceptual understanding [11], [12]. These approaches often result in algorithmic performance without genuine reasoning [13] and lack relevance to students' everyday experiences and practical applications [14]. Given these limitations, there is an urgent need to investigate innovative instructional methods and frameworks.

A range of strategies has been proposed to address challenges in students' learning of mathematical concepts [15]. Structured teaching approaches have been shown to promote critical thinking and foster deep conceptual understanding [16]. These methods provide systematic, step-by-step guidance for problem-solving, creating a supportive environment that reduces anxiety and builds confidence [17], [18]. Implementing teaching frameworks that methodically nurture conceptual understanding is one effective strategy [19]. Techniques such as guided inquiry, collaborative learning, and scaffolded instruction enhance cognitive engagement and performance [20], [21]. Instructional frameworks like problem-based learning and blended learning models, particularly when combined with probing-prompting strategies, have demonstrated success in improving students' conceptual understanding and mathematical reasoning [22]-[26]. Despite growing interest in structured teaching approaches, there is limited research on the direct impact of the AIC learning model on students' conceptual understanding in geometry.

The AIC learning model comprises three stages: Analyze, Identify, and Connect. This model centers on student engagement and aims to improve conceptual understanding by emphasizing active participation in the learning process rather than passive information reception [27]. Through the AIC model, students independently explore concepts and problems, fostering understanding beyond rote memorization [28]. The framework is informed by the Abstraction in Context (AiC) learning model developed by Dreyfus and Kidron [29], which introduces the Analyze-Identify-Connect structure by modeling abstraction in three phases: Need, Emergence, and Consolidation. Additionally, the RBC Model by Hershkowitz et al. [30] identifies epistemic actions—Recognizing, Building-with, and constructing—that correspond to the micro-level behaviors observed during the Analyze and Identify stages of the AIC model. Together, these frameworks underpin the three-step Analyze-Identify-Connect process.

Thus, this study centers on how the implementation of the AIC learning model directly influences students' cognitive processes and, consequently, their conceptual understanding in geometry. By structuring instruction into the Analyze, Identify, and Connect stages, the AIC model actively guides students through processes of abstraction, pattern recognition, and connection-making, which are fundamental to deep conceptual learning. This approach moves students beyond memorization to engage in critical analysis, comparison, and synthesis of geometric concepts—cognitive actions that are often underdeveloped in conventional teaching environments [9], [31]. As students participate in the scaffolded activities of the AIC model, they internalize mathematical relationships and learn to interpret, represent, and apply geometric ideas more flexibly [28], [32]. Thus, the AIC framework creates a causal pathway in which its structured, interactive approach fosters key cognitive processes that result in measurable gains in conceptual understanding.

2. RESEARCH METHOD

2.1 Research Design

This research employs a quasi-experimental method, utilizing a non-equivalent control group structure. It is the most appropriate design for the objectives of this study because it allows for the comparison of outcomes between the two groups. This design strikes a balance between methodological and practical feasibility, making it well-suited for evaluating the effectiveness of teaching innovations in authentic educational environments. The experimental group was implemented with the AIC learning model, while the control group was implemented with the conventional teaching approach. The design can be seen in the Table 1.

Table 1. Treatment of

Groups	Pre-test	Treatment	Post-test
Experimental	√	√	√
Control	√	-	√

√: With the treatment of the AIC learning model

-: Without the treatment of the AIC learning model

2.2 Population and Sample

The study's subjects comprised grade nine junior high school students. Out of the entire population of grade nine, 95 students were selected using a purposive sampling method, who are at a cognitive developmental stage conducive to abstract reasoning and conceptual learning, making them ideal for evaluating the AIC learning model's effectiveness on mathematical understanding. Purposive sampling was utilized to select students, as it aims to capture a representative group that meets specific criteria relevant to the research objectives. The steps of the AIC learning model are presented in Figure 1.

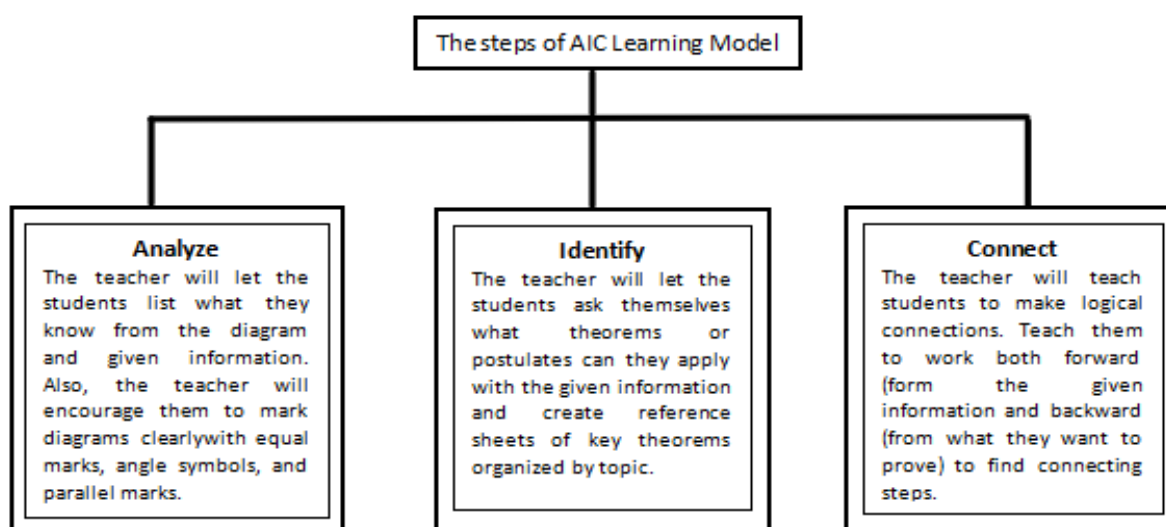


Figure 1. Steps of the AIC Learning Model

2.3 Data Collection Instrument

The study utilized a test questionnaire to assess students' conceptual understanding, coupled with an observation checklist to evaluate the applicability of the learning model [33]. Prior to administering the test questionnaire, the researchers prepared 40 items for the tests, which included topics in geometry. Items were formulated based on the National Council for Teachers in Mathematics (NCTM) principles and standards to assess students' conceptual understanding of mathematical concepts. Table 2 below shows the conceptual understanding indicators according to NCTM where the test items for conceptual understanding in geometry were specifically designed to assess key cognitive indicators, including defining concepts verbally and in writing, making examples and non-examples, using various symbols, transforming representations, identifying concept properties, comparing concepts, and interpreting concepts.

Table 2. Conceptual understanding indicators

Indicators
1. Defining concepts verbally and in writing
2. Making examples and not examples
3. Using various symbols to present a concept
4. Change the form of representation to various forms
5. Identify the nature of a concept
6. Compare various concepts
7. Interpret concepts

The researcher had also tested the validity of the test items for the conceptual understanding test. Experts in the field of mathematics lent their time to validate the instrument and provided valuable feedback. After thorough validation, the test questionnaire was refined, and items were reduced. The researcher then conducted pilot testing of the test questionnaire on a different population than the target population to assess the appropriateness of the questions. Several questions were deleted as they needed to be rejected based on the

results of the item analysis. The researcher then proceeded with the computation of the reliability index for conceptual understanding test results, which resulted in Cronbach's Alpha of .82. The final examination tool encompasses nineteen questions targeting conceptual understanding of students in geometry lessons.

2.4 Data Analysis Techniques

In the analysis of data, the researcher utilized the t-test to analyze differences in the pre-test and post-test results. The t-test is a statistical technique used to quantify the difference between the means of a variable from up to two samples. It also identifies if there exists a notable statistical variance between them. The choice of using the t-test in this study was based on the premise of data being regular and homogeneous [34]. Before diving deep into the analysis of the data, the test for assumptions was verified as to normality and homogeneity using statistical software SPSS, obtaining a significance threshold above 0.05.

The effect size was calculated to assess the magnitude of variance between pre-test and post-test outcomes. The researcher included effect size calculations to quantify the magnitude of the difference between the experimental and control groups. In educational research, effect sizes provide practical interpretation beyond statistical significance, indicating the method's effect on students' conceptual understanding and suggesting the intervention produced educationally meaningful gains. This statistical and practical framework strengthens the validity and applicability of the study's findings within the educational context [35]. The analysis of effect size facilitated the evaluation of whether the observed progress in final test outcomes resulted from the implementation of the AIC learning model.

2.5 Research Procedure

The researcher carefully followed the ethical procedures throughout the study. During preparation and planning, the researcher prepared all necessary documents and secured permission from the School Administrator to conduct the study. The researcher used random selection to determine which class would serve as the experimental and control groups. The researcher did not inform the students that they were subject to an experiment to avoid the Hawthorne effect.

During the pre-test administration, the researcher used a specifically designed questionnaire and assessment tool to assess students' conceptual understanding. The experimental and control groups were given the pre-test at different times to ensure fairness. The researcher ensured conducive testing conditions by providing a quiet environment, sufficient time to answer the test, and clear instructions for the test.

In the intervention phase, the experiment began in the next class meeting. The experimental group was exposed to the AIC learning model. In contrast, the control group continued with the conventional teaching method. Each class session for both groups began with preliminary activities: a prayer, attendance checks, a review of previous lessons, and a warm-up. The treatment is carried out only on the learning model, in terms of assigning activities and topics taught, with each session lasting one hour. All students across both groups were instructed by the same teacher and were provided with identical learning materials and mathematics problems. This experiment lasted several meetings that covered all the lessons on the geometry topic, and after the experiment, a post-test was administered to both groups.

During the post-test administration, the researcher administered the same questionnaire and assessment tools to both groups to measure their learning gains. Testing conditions were maintained, and completed tests were securely stored. Subsequently, the researcher tabulated the collected data and proceeded with data analysis. The flow during the implementation stage is highlighted in Figure 2.

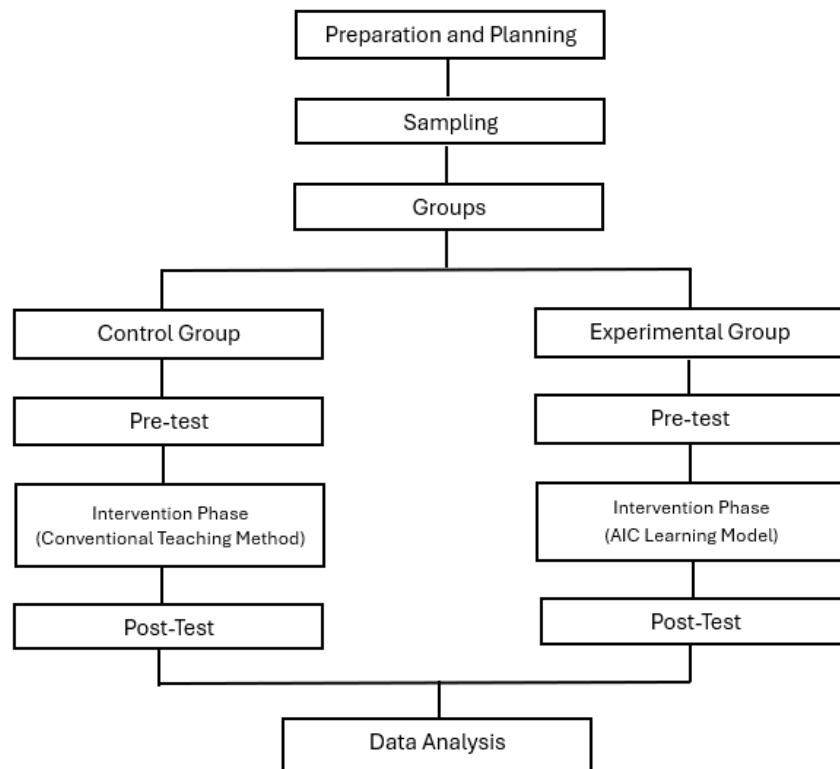


Figure 2. Research procedure chart

3. RESULTS AND DISCUSSION

3.1 Test Results on Students' Conceptual Understanding

Student conceptual understanding was evaluated using pre-test and post-test assessments, as shown in Table 3. The primary aim was to assess whether the learning model implemented in the experimental group was more effective than the conventional instructional method used in the control group for enhancing conceptual understanding and overall mathematics achievement. In the pre-test, the experimental group achieved a mean score of 4.95, while the control group averaged 4.88. In the post-test, the experimental group attained a mean score of 14.15, compared to 11.78 for the control group. These results indicate a clear improvement in scores for both groups from pre-test to post-test. Notably, the experimental group demonstrated a greater increase in mean score compared to the control group.

Table 3. Pre-test and Post-test results of students' conceptual understanding

Outcomes	Control		Experimental	
	Pre-test	Post-test	Pre-test	Post-test
Mean	4.88	11.78	4.95	13.15
Max Score	8	13	9	16
Min Score	3	4	4	6

The substantial gains observed in the experimental group's post-test scores reflect not only the effectiveness of the AIC learning model but also its impact on students' cognitive processes. The greater increase in mean scores suggests that the AIC framework successfully guided students through deeper stages of understanding, such as abstraction, connection-making, and conceptual restructuring. These results indicate that students exposed to the AIC model moved beyond memorizing procedures and began to internalize and apply geometric concepts with greater flexibility and accuracy. The statistical improvement thus represents a shift toward higher-order thinking, supporting the conclusion that structured, constructivist-based instruction like AIC can meaningfully enhance students' conceptual development in mathematics.

3.2 N-Gain Values of Both Groups

Table 4 indicates a more pronounced score enhancement in the experimental group compared to the control group, as evidenced by the data gathered from both classes. The experimental group achieved an N-

gain score of 0.74 for conceptual understanding, while the control group secured a score of 0.62. The results show that the experimental group achieved a higher N-gain score, indicating they achieved 74% of the potential learning gain, compared to the control group, which achieved 62%. This disparity between the experimental and control groups underscores the efficacy of the AIC learning model in students' conceptual understanding.

Table 4. N-gain values of the conceptual understanding test results

Outcome	N-gains	Classification Criteria
Control	.62	Moderate
Experimental	.74	High Gain

This suggests that the AIC model not only accelerated knowledge acquisition but also deepened cognitive processes such as analysis, synthesis, and knowledge transfer—key components of conceptual understanding. Educationally, this disparity highlights the model's capacity to foster more effective cognitive engagement, enabling students to make meaningful connections between geometric concepts and apply them in novel contexts. The data thus provide strong evidence that the AIC framework promotes not just higher achievement, but also more profound and lasting changes in students' mathematical cognition.

3.3 Effect size

From the effect size test results shown in Table 5, the AIC learning model effectively increases students' conceptual understanding. It displays the effect size in students' conceptual understanding to obtain a score of 0.83, which falls into the moderate category, indicating that conceptual understanding positively impacts mathematics subjects [36]. This is because the steps in the AIC learning model encourage students to think at a higher level using their analytical skills.

Table 5. Test of Effect Size

Variable	Effect Size	S. D	Criteria
Conceptual Understanding	.83	1.32	Moderate

The test results demonstrate that the AIC learning model enhances students' conceptual understanding in geometry. This instructional framework is also effective in improving mathematics achievement, with success attributable to the model's structured steps. Each step of the AIC learning model is critical for developing students' understanding and reasoning. Thus, mathematics educators should closely examine the model's implementation to achieve positive outcomes.

Table 6 provides a step-by-step guide to the AIC model, including corresponding actions and indicators. The model comprises three distinct stages. The first stage is analysis, in which students examine the provided diagrams, figures, statements, or information in the problem. This stage requires careful observation and a thorough breakdown of the given information. The second stage is identification, where students determine the applicable theorems or postulates relevant to solving the problem. Students classify and organize the theorems that connect the given information to the desired conclusion. The third stage is connection, where students establish a logical sequence. In this stage, the conclusion of one step serves as the premise for the next, enabling students to progress from the initial statements to the final proof by integrating theorems and definitions.

Table 6. The flow of the AIC learning model

Steps	Action	Indicators	Conceptual Enhancement
Analyze This step focuses on careful observation of the problem or figure [29][[30].	Teachers will let the students <i>analyze</i> the diagram/figure/ problem text	<ul style="list-style-type: none"> Students can break down a theorem or word problem to identify given information and required outcomes Students can use dynamic geometry software or physical manipulatives to explore and manipulate a figure. 	<ul style="list-style-type: none"> Develops visualization and helps students distinguish relevant geometric properties from irrelevant perceptual details [37] Fosters abstract reasoning by translating complex textual information into clear, verifiable geometric components [29], [30].
Identify This step requires	The teacher will direct students to	<ul style="list-style-type: none"> Students can name the shapes, 	<ul style="list-style-type: none"> Promotes analysis by requiring them to use

students to classify and categorize the features they have analyzed, linking them to formal geometric definitions [29][30].	<i>identify</i> relevant concepts or necessary tools	theorems, or axioms involved. <ul style="list-style-type: none"> Students can determine the formula or prior knowledge required. 	precise geometric vocabulary and apply formal definitions, moving them past visual recognition [37]. <ul style="list-style-type: none"> Builds a procedural-conceptual link, ensuring that a procedure is chosen because the underlying concept demands it, not just because a rule was memorized [38], [39].
Connect This is the most critical step for achieving conceptual understanding, as it moves the student from rote facts to conceptual mastery [29].	The teacher will direct students to <i>connect</i> within the concept, prior knowledge, or solution/proof	<ul style="list-style-type: none"> Students can link multiple representations of the exact figure. Students can use analogy to link a new concept to a familiar one. Students can synthesize the identified concepts and properties into a logical sequence of steps that forms a proof or leads to a solution. 	<ul style="list-style-type: none"> Cultivates relational understanding, allowing students to see the same concept from multiple perspectives, which is essential for transferability [39], [29], [40]. Reinforces the abstract structure of geometry, allowing students to construct rigorous proofs and fully grasp the <i>why</i> behind a solution, rather than just the <i>how</i> [38]-[40].

Conceptual understanding represents meaningful learning, enabling students to transfer knowledge efficiently and solve mathematical problems. The AIC teaching framework was shown to enhance students' conceptual understanding. Implementing the AIC model supports the development of analytical skills based on acquired information, thereby deepening learners' understanding. In contrast, the explicit teaching method offers more direct teacher guidance, which may reduce opportunities for independent analytical skill development.

The findings of this study are consistent with previous research that demonstrates a significant positive relationship between guided practice and conceptual understanding. Structured instruction frequently results in guided practice, which improves students' ability to transfer knowledge to new problems by promoting understanding of underlying concepts rather than focusing solely on procedural knowledge [41]. Moreover, Mutawah and Baddar [22], Hiebert and Carpenter [39] found a significant positive correlation between guided practice and conceptual understanding. Structured instruction often translates into guided practice, significantly developing students' ability to transfer knowledge to new problems because they understand the *why* (the concept) rather than just the *how* (the procedure). This approach involves a careful blend of direct teaching and active student engagement. These findings are also consistent with the works of Hiebert and Carpenter [39], who pointed out that conceptual understanding is knowledge that is connected, coherent, and part of a network, building a more robust and interconnected cognitive structure, which is essential for proper understanding. Also, in a meta-analysis by Freeman et al. [42] it was shown that active learning strategies consistently lead to higher achievement than conventional lecture formats.

By guiding students through the Analyze, Identify, and Connect stages, the AIC framework creates structured opportunities for active knowledge construction, abstraction, and the making of meaningful connections. These processes are central to constructivist perspectives, which emphasize that learners build understanding by engaging with content, reflecting on experiences, and connecting new concepts to existing cognitive frameworks. The study shows that the AIC model fosters the development of conceptual networks, moving students beyond rote memorization toward deeper comprehension and the ability to transfer knowledge to unfamiliar contexts.

The findings also underscore the practical value of integrating the AIC model into mathematics instruction. The structured and interactive nature of AIC not only enhances student engagement and analytical thinking but also offers teachers a scaffolded approach to lesson design that promotes independent reasoning and conceptual mastery. By encouraging students to analyze, represent, and connect mathematical ideas, the AIC framework equips mathematics educators with strategies to address persistent gaps in conceptual understanding

and cultivate lifelong mathematical literacy. These implications are particularly relevant for teachers seeking to move beyond procedural teaching and foster a classroom environment grounded in active, student-centered learning.

4. CONCLUSION

This study found that the AIC learning model effectively improved students' conceptual understanding of geometry, as evidenced by greater gains and effect size. This study also advances understanding of the effect of structured teaching approach on cognitive development in mathematics. Mathematics educators are also encouraged to adopt this model more broadly, given its effectiveness in improving student engagement and analytical thinking skills, and further research is recommended to validate these results across broader educational contexts.

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REFERENCES

- [1] Ç. Biber, A. Tuna, and S. Korkmaz, "The eighth-grade students' mistakes and misconceptions on the subject of angles," *Eur. J. Sci. Math. Educ.*, vol. 1, no. 2, pp. 50–59, 2013. doi: 10.30935/scimath/9387.
- [2] K. K. Poon and C. K. Leun, "A study of geometry understanding via logical reasoning in Hong Kong," *Int. J. Math. Teach. Learn.*, vol. 17, no. 3, pp. 93–123, 2016.
- [3] A. Özerem, "Misconceptions in geometry and suggested solutions for seventh-grade students," *Procedia-Social Behav. Sci.*, vol. 55, pp. 720–729, 2012. doi: 10.1016/j.sbspro.2012.09.557.
- [4] R. Mangarin and D. Caballes, "Difficulties in learning mathematics: A systematic review," *Int. J. Res. Sci. Innov.*, vol. 11, pp. 401–405, 2024. doi: 10.51244/IJRSI.2024.1109037.
- [5] M. Al-kassab, "Mathematics learning challenges and difficulties: A student's perspective," in *Mathematics challenges and difficulties: A students' perspective*, M. Al-kassab, Ed. Springer, 2023, p. 418. doi: 10.1007/978-981-99-0447-1_27.
- [6] T. Tambychika and T. S. M. Meerah, "Students' difficulties in mathematics problem-solving: What do they say?," *Proc. Int. Conf. Math. Educ. Res. 2010 (ICMER 2010)*, 2010.
- [7] M. Ncube and K. Luneta, "Concept-based instruction: Improving learner performance in mathematics through conceptual understanding," *Pythagoras*, vol. 46, no. 1, Art. a815, 2025. doi: 10.4102/pythagoras.v46i1.815.
- [8] J. Enu, O. K. Agyman, and D. Nkum, "Factors influencing students' mathematics performance in some selected colleges of education in Ghana," *Int. J. Educ. Learn. Develop.*, vol. 3, no. 3, pp. 68–74, 2015. doi: 10.13189/ujer.2019.070913.
- [9] Y. Suleiman and A. Hammed, "Perceived causes of students' failure in mathematics in Kwara State junior secondary schools: implications for educational managers," *Int. J. Educ. Stud. Math.*, vol. 6, no. 1, pp. 19–33, 2019.
- [10] I. Vale and A. Barbosa, "Active learning strategies for effective mathematics teaching and learning," *Eur. J. Sci. Math. Educ.*, vol. 11, no. 3, pp. 573–588, 2023. doi: 10.30935/scimath/13135.
- [11] S. Lubienski, "A second look at mathematics achievement gaps: Intersections of race, class, and gender in NAEP data." Presented at American Educational Research Association, Seattle, WA, United States, Apr. 2001.
- [12] M. Strutchens and E. A. Silver, "Students' performance, school experiences, and attitudes and beliefs," in *Results from the Seventh Mathematics Assessment of the National Assessment of Educational Progress*, E. A. Silver and P. A. Kenney, Eds. The National Council of Teachers of Mathematics, 2000, pp. 45–72.
- [13] A. Schoenfeld, "On mathematics as sense-making: An informal attack on the unfortunate divergence of formal and informal mathematics," in *Informal Reasoning and Education*, J. F. Coss, D. N. Perkins, and J. W. Segal, Eds. 1992, pp. 311–344.
- [14] J. Marshal, "Math wars 2: It is the teaching, stupid!," *Phi Delta Kappan*, vol. 87, no. 5, pp. 356–363, 2006.
- [15] A. Agustyaningrum, Y. Hanggara, A. Husna, A. M. Abadi, and A. Mahmudii, "An analysis of students' mathematical reasoning ability in an abstract algebra course," *Int. J. Sci. Technol. Res.*, vol. 8, no. 12, pp. 2800–2805, Dec. 2019. doi: 10.13189/ujer.2020.080313.
- [16] B. J. Zimmerman, D. H. Schunk, and M. K. DiBenedetto, "The role of self-efficacy and related beliefs in self-regulation of learning and performance," in *Handbook of Competence and Motivation: Theory and Application*, 2nd ed., A. J. Elliot, C. S. Dweck, and D. S. Yeager, Eds. The Guilford Press, 2017, pp. 313–333.
- [17] D. A. Housel, "Reflections on preservice preparation and professional development among instructors of adult emergent bi/multilingual learners," Ph.D. dissertation, City Univ. of New York, New York, NY, 2021.
- [18] L. P. Aujero and R. S. Fuentebilla, "Structured approach in critical thinking skills and mathematics anxiety of grade 12 STEM students of Isulan National High School," *Int. J. Res. Innov. Soc. Sci.*, vol. 9, no. 4, pp. 260–269, 2025. doi: 10.47772/IJRIS.2025.90400021.
- [19] L. A. Rohmah, B. S. Anggoro, and W. Gunawan, "PDEODE strategy assisted by GeoGebra: improving students' critical thinking and mathematical analysis," *Online Learn. Educ. Res.*, vol. 3, no. 1, pp. 15–22, 2023. doi: 10.58524/oler.v3i1.203.

- [20] M. A. Basir and H. R. Maharani, "Reasoning ability of students in solving mathematical problems viewed from cognitive style," in *The 2nd International Seminar on Educational Technology*, vol. 99, 2016, pp. 99–102. doi: 10.1007/978-3-319-45583-7_11.
- [21] L. S. Vygotsky, *Mind in Society: The Development of Higher Psychological Processes*. Harvard University Press, 1978.
- [22] M. A. Mutawah and F. M. Baddar, "Teaching approaches and students' conceptual understanding in geometry," [Publisher name unavailable], 2019.
- [23] N. Ariyana and F. G. Putra, "SiMaYang type II learning model assisted by Kahoot application: its impact on improving students' concept understanding based on APOS theory," *Online Learn. Educ. Res.*, vol. 1, no. 1, pp. 17–23, 2021. doi: 10.58524/oler.v1i1.9.
- [24] A.-L. Tan, Y. S. Ong, Y. S. Ng, and J. H. J. Tan, "STEM problem solving: Inquiry, concepts, and reasoning," *Sci. Educ.*, vol. 32, no. 2, pp. 381–397, 2023. doi: 10.1007/s11191-021-00310-2.
- [25] V. N. Saputri and A. D. Kesumawardani, "Problem-based learning (PBL) model: how does it influence metacognitive skills and independent learning?," *J. Adv. Sci. Math. Educ.*, vol. 1, no. 1, pp. 27–32, 2021. doi: 10.58524/jasme.v1i1.18.
- [26] N. Phumeechanya and P. Wannapiroon, "Ubiquitous Scaffold Learning Environment Using Problem-based Learning to Enhance Problem-solving Skills and Context Awareness," 2014.
- [27] F. E. Men, B. Gunur, R. Jundu, and P. Raga, "Critical thinking profiles of junior high school students in solving plane geometry problems based on cognitive style and gender," *Indones. J. Sci. Math. Educ.*, vol. 3, no. 2, pp. 237–244, 2020. doi: 10.24042/ij sme.v3i2.5955.
- [28] F. G. Putra *et al.*, "Enhancing mathematical reasoning: role of the search, solve, create, and share learning," *J. Educ. Learn. (EduLearn)*, vol. 18, no. 3, pp. 967–975, 2024. doi: 10.11591/edulearn.v18i3.21399.
- [29] T. Dreyfus and A. Kidron, "Introduction to Abstraction in Context (AiC)," in *Thematic Working Group 19: Proof and Proving in Mathematics Education*, M. Artigue, M. F. C. Leal, T. Dreyfus, and L. T. De Oliveira, Eds. 2014. doi: 10.1007/978-3-319-17187-6_7.
- [30] R. Hershkowitz, B. B. Schwarz, and T. Dreyfus, "Abstraction in context: epistemic actions," *J. Res. Math. Educ.*, vol. 32, no. 2, pp. 195–218, 2001.
- [31] E. Gutstein, "The real world as we have seen it: Latino/a parents' voices on teaching mathematics for social justice," *Math. Think. Learn.*, vol. 8, no. 3, pp. 331–358, 2006.
- [32] A. Son, "The students' abilities on mathematical connections: A comparative study based on learning models intervention," *Jurnal Pendidikan dan Pembelajaran Matematika*, vol. 14, pp. 72–87, 2022.
- [33] A. A. I. Y. Pramawati, I. G. P. A. Pramerta, and D. G. B. Erawan, "Enhancing translation education with RATPRO: A CAT tools-based learning model," *Eur. Res. J. Educ. English*, vol. 12, no. 2, 2024. doi: 10.25134/erjee.v12i2.9097.
- [34] R. Darmayanti, M. Syaifuddin, N. Rizki, R. Sugianto, and N. Hasanah, "High school students' mathematical representation ability: Evaluation of disposition based on mastery learning assessment model (MLAM)," *J. Adv. Sci. Math. Educ.*, vol. 2, no. 1, Art. 1, 2022. doi: 10.58524/jasme.v2i1.93.
- [35] A. L. Robin, "Incorporating Concept Programs into Study Guides in a Personalized Instruction Course," *Teach. Psychol.*, 1980. doi: 10.1207/s15328023top0701_7.
- [36] T. W. Oktaviani *et al.*, "Comparison Analysis of the Effectiveness of Three Microcontroller Practicum Media (Trainer-Box, Simulation Software, and Breadboard Hands-On) at SMK Negeri 1 Sentani," *Riwayat Educ. J. Hist. Hum.*, 2025. doi: 10.24815/jr.v8i4.49722.
- [37] D. Fuys, D. Geddes, and R. Tischler, *The Van Hiele Model of Thinking in Geometry Among Adolescents*. National Council of Teachers of Mathematics (NCTM), 1988.
- [38] B. Rittle-Johnson and M. Schneider, "Developing conceptual and procedural knowledge in mathematics," *Oxford Handbooks Online*, 2015.
- [39] J. Hiebert and T. P. Carpenter, "Learning and teaching with understanding," in *Handbook of Research on Mathematics Teaching and Learning*, D. A. Grouws, Ed. Macmillan Publishing Company, 1992, pp. 65–97.
- [40] National Council of Teachers of Mathematics, *Principles and Standards for School Mathematics*. NCTM, 2000.
- [41] S. P. Simkins and M. H. Maier, "The first course in economics: A look at what has changed and what should change," *J. Econ. Educ.*, vol. 39, no. 3, pp. 227–245, 2008.
- [42] S. Freeman *et al.*, "Active learning increases student performance in science, engineering, and mathematics," *Proc. Natl. Acad. Sci.*, vol. 111, no. 23, pp. 8410–8415, Jun. 2014. doi: 10.1073/pnas.1319030111.